25.1. Model: Balmer's formula predicts a series of spectral lines in the hydrogen spectrum.
Solve: Substituting into the formula for the Balmer series,

$$\lambda = \frac{91.18 \text{ nm}}{\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)} \Rightarrow \lambda = \frac{91.18 \text{ nm}}{\dfrac{1}{2^2} - \dfrac{1}{6^2}} = 410.3 \text{ nm}$$

where n = 3, 4, 5, 6, … and where we have used n = 6. Likewise for n = 8 and n = 10, $\lambda = 389.0$ nm and $\lambda = 379.9$ nm.

25.2. Model: Balmer's formula predicts a series of spectral lines in the hydrogen spectrum.
Solve: Balmer's formula is

$$\lambda = \frac{91.18 \text{ nm}}{\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)} \qquad n = 3, 4, 5, 6, \ldots$$

As $n \to \infty$, $1/n^2 \to 0$. Thus, $\lambda_{n\to\infty} = 4(91.18 \text{ nm}) = 364.7$ nm.

25.3. Model: Balmer's formula predicts a series of spectral lines in the hydrogen spectrum.
Solve: Using Balmer's formula,

$$\lambda = 389.0 \text{ nm} = \frac{91.18 \text{ nm}}{\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)} \Rightarrow \frac{1}{4} - \frac{1}{n^2} = 0.2344 \Rightarrow n = 8$$

25.4. Model: The angles of incidence for which diffraction from parallel planes occurs satisfy the Bragg condition.

Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where m = 1, 2, 3, … For first and second order diffraction,

$$2d\cos\theta_1 = (1)\lambda \qquad 2d\cos\theta_2 = (2)\lambda$$

Dividing these two equations,

$$\frac{\cos\theta_2}{\cos\theta_1} = 2 \Rightarrow \theta_2 = \cos^{-1}(2\cos\theta_1) = \cos^{-1}(2\cos 68°) = 41°$$

25.5. Model: The angles of incidence for which diffraction from parallel planes occurs satisfy the Bragg condition.

Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$. For $m = 1$ and for two different wavelengths,

$$2d\cos\theta_1 = (1)\lambda_1 \qquad 2d\cos\theta_1' = (1)\lambda_1'$$

Dividing these two equations,

$$\frac{\cos\theta_1'}{\cos\theta_1} = \frac{\lambda_1'}{\lambda_1} \Rightarrow \frac{\cos\theta_1'}{\cos 54°} = \frac{0.15 \text{ nm}}{0.20 \text{ nm}} \Rightarrow \theta_1' = \cos^{-1}(0.4408) = 64°$$

25.6. **Model:** The angles corresponding to the various orders of diffraction satisfy the Bragg condition.
Solve: The Bragg condition for $m = 1$ and $m = 2$ gives

$$2d\cos\theta_1 = (1)\lambda \qquad 2d\cos\theta_2 = (2)\lambda$$

Dividing these two equations,

$$\cos\theta_1 = \frac{\cos\theta_2}{2} = \frac{\cos 45°}{2} \Rightarrow \theta_1 = \cos^{-1}\left(\frac{\cos 45°}{2}\right) = 69.3°$$

25.7. Model: The angles corresponding to the various diffraction orders satisfy the Bragg condition.

Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where m = 1, 2, 3, … The maximum possible value of m is the number of possible diffraction orders. The maximum value of $\cos\theta_m$ is 1. Thus,

$$2d = m\lambda \Rightarrow m = \frac{2d}{\lambda} = \frac{2(0.180 \text{ nm})}{(0.085 \text{ nm})} = 4.2$$

We can observe up to the fourth diffraction order.

25.8. **Model:** Use the photon model of light.

Solve: The energy of the photon is

$$E_{\text{photon}} = hf = h\frac{c}{\lambda} = \left(6.63\times10^{-34}\text{ Js}\right)\left(\frac{3.0\times10^{8}\text{ m/s}}{500\times10^{-9}\text{ m}}\right) = 3.98\times10^{-19}\text{ J}$$

Assess: The energy of a single photon in the visible light region is extremely small.

25.9. Model: Use the photon model of light.

Solve: The energy of the single photon is

$$E_{\text{photon}} = hf = h\left(\frac{c}{\lambda}\right) = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)\left(3.0\times10^{8}\ \text{m/s}\right)}{1.0\times10^{-6}\ \text{m}} = 1.99\times10^{-19}\ \text{J}$$

$$\Rightarrow E_{\text{mol}} = N_{\text{A}}E_{\text{photon}} = \left(6.023\times10^{23}\right)\left(1.99\times10^{-19}\ \text{J}\right) = 1.2\times10^{5}\ \text{J}$$

Assess: Although the energy of a single photon is very small, a mole of photons has a significant amount of energy.

25.10. **Model:** Use the photon model of light.

Solve: The energy of a photon with wavelength λ_1 is $E_1 = hf_1 = hc/\lambda_1$. Similarly, $E_2 = hc/\lambda_2$. Since E_2 is equal to $2E_1$,

$$\frac{hc}{\lambda_2} = 2\frac{hc}{\lambda_1} \Rightarrow \lambda_2 = \frac{\lambda_1}{2} = \frac{600 \text{ nm}}{2} = 300 \text{ nm}$$

Assess: A photon with λ = 300 nm has twice the energy of a photon with λ = 600 nm. This is an expected result, because energy is inversely proportional to the wavelength.

25.11. Model: Use the photon model of light.

Solve: The energy of the x-ray photon is

$$E = hf = h\left(\frac{c}{\lambda}\right) = \left(6.63\times10^{-34}\ \text{Js}\right)\left(\frac{3.0\times10^{8}\ \text{m/s}}{1.0\times10^{-9}\ \text{m}}\right) = 2.0\times10^{-16}\ \text{J}$$

Assess: This is a very small amount of energy, but it is larger than the energy of a photon in the visible wavelength region.

25.12. Solve: Your mass is, say, $m \approx 70$ kg and your velocity is 1 m/s. Thus, your momentum is $p = mv \approx (70$ kg)(1 m/s) = 70 kg m/s. Your de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\ \text{Js}}{70\ \text{kg m/s}} \approx 9 \times 10^{-36}\ \text{m}$$

25.13. Solve: **(a)** The baseball's momentum is $p = mv = (0.200\text{ kg})(30\text{ m/s}) = 6.0\text{ kg m/s}$. The baseball's de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\text{ Js}}{6.0\text{ kg m/s}} = 1.1\times10^{-34}\text{ m}$$

(b) Using $\lambda = h/p = h/mv$, we have

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\text{ Js}}{(0.200\text{ kg})(0.20\times10^{-9}\text{ m})} = 1.7\times10^{-23}\text{ m/s}$$

25.14. Visualize: We'll employ Equations 25.8 $(\lambda = h/p)$ and 25.9 $(E = p^2/2m)$ to express the wavelength in terms of kinetic energy.

Solve: First solve Equation 25.9 for p: $p = \sqrt{2mE}$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}\,\mathrm{J\cdot s}}{\sqrt{2(9.11\times10^{-31}\ \mathrm{kg})(2.4\times10^{-19}\,\mathrm{J})}} = 1.0\ \mathrm{nm}$$

Assess: The energy given is about 1.5 eV, which is a reasonable amount of energy. The resulting wavelength is a few to a few dozen times the size of an atom.

25.15. **Solve:** **(a)** For an electron, the momentum $p = mv = (9.11\times10^{-31}\ \text{kg})v$. The de Broglie wavelength is

$$\lambda = \frac{h}{p} = 0.20\times10^{-9}\ \text{m} \Rightarrow 0.20\times10^{-9}\ \text{m} = \frac{6.63\times10^{-34}\ \text{Js}}{(9.11\times10^{-31}\ \text{kg})v} \Rightarrow v = 3.6\times10^{6}\ \text{m/s}$$

(b) For a proton, $p = mv = (1.67\times10^{-9}\ \text{kg})v$. The de Broglie wavelength is

$$\lambda = \frac{h}{p} = 0.20\times10^{-9}\ \text{m} \Rightarrow 0.20\times10^{-9}\ \text{m} = \frac{6.63\times10^{-34}\ \text{Js}}{(1.67\times10^{-27}\ \text{kg})v} \Rightarrow v = 2.0\times10^{3}\ \text{m/s}$$

25.16. **Model:** The momentum of a wave-like particle has discrete values given by $p_n = n(h/2L)$ where $n =$ 1, 2, 3, ….

Solve: Because we want the smallest box and the momentum of the electron can not exceed a given value, n must be minimum. Thus,

$$p_1 = mv = \frac{h}{2L} \Rightarrow L = \frac{h}{2mv} = \frac{6.63\times10^{-34}\ \text{Js}}{2(9.11\times10^{-31}\ \text{kg})(10\ \text{m/s})} = 0.036\ \text{mm}$$

25.17. Model: A confined particle can have only discrete values of energy.
Solve: From Equation 24.14, the energy of a confined electron is

$$E_n = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots$$

The minimum energy is

$$E_1 = \frac{h^2}{8mL^2} \Rightarrow L = \frac{h}{\sqrt{8mE_1}} = \frac{6.63\times10^{-34}\ \text{Js}}{\sqrt{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.5\times10^{-18}\ \text{J}\right)}} = 2.0\times10^{-10}\ \text{m} = 0.20\ \text{nm}$$

25.18. Model: Model the 5.0-fm-diameter nucleus as a box of length $L = 5.0$ fm.
Solve: The proton's energy is restricted to the discrete values

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)^2 n^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(5.0\times10^{-15}\ \text{m}\right)^2} = \left(1.316\times10^{-12}\ \text{J}\right)n^2$$

where $n = 1, 2, 3, \ldots$ For $n = 1$, $E_1 = 1.3\times10^{-12}\ \text{J}$, for $n = 2$, $E_2 = \left(1.316\times10^{-12}\ \text{J}\right)4 = 5.3\times10^{-12}\ \text{J}$, and for $n = 3$, $E_3 = 9E_1 = 1.2\times10^{-11}\ \text{J}$.

25.19. Model: The generalized formula of Balmer predicts a series of spectral lines in the hydrogen spectrum.

Solve: **(a)** The generalized formula of Balmer

$$\lambda = \frac{91.18 \text{ m}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

with m = 1 and n > 1 accounts for a series of spectral lines. This series is called the Lyman series and the first two members are

$$\lambda_1 = \frac{91.18 \text{ m}}{\left(1 - \frac{1}{2^2}\right)} = 121.6 \text{ nm} \qquad \lambda_2 = \frac{91.18 \text{ nm}}{\left(1 - \frac{1}{3^2}\right)} = 102.6 \text{ nm}$$

For $n = 4$ and $n = 5$, $\lambda_3 = 97.3$ nm and $\lambda_4 = 95.0$ nm .

(b) The Lyman series converges when $n \to \infty$. This means $1/n^2 \to 0$ and $\lambda \to 91.18$ nm.

(c) For a diffraction grating, the condition for bright (constructive interference) fringes is $d \sin\theta_p = p\lambda,$ where p = 1, 2, 3, … For first-order diffraction, this equation simplifies to $d\sin\theta = \lambda.$ For the first and second members of the Lyman series, the above condition is $d\sin\theta_1 = \lambda_1 = 121.6$ nm and $d\sin\theta_2 = \lambda_2 = 102.6$ nm. Dividing these two equations yields

$$\sin\theta_2 = \left(\frac{102.6 \text{ nm}}{121.6 \text{ nm}}\right)\sin\theta_1 = (0.84375)\sin\theta_1$$

The distance from the center to the first maximum is $y = L\tan\theta.$ Thus,

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.376 \text{ m}}{1.5 \text{ m}} \Rightarrow \theta_1 = 14.072° \Rightarrow \sin\theta_2 = (0.84375)\sin(14.072°) \Rightarrow \theta_2 = 11.84°$$

Applying the position formula once again,

$$y_2 = L\tan\theta_2 = (1.5 \text{ m})\tan(11.84°) = 0.314 \text{ m} = 31.4 \text{ cm}$$

25.20. **Model:** The generalized formula of Balmer predicts a series of spectral lines in the hydrogen spectrum.

Solve: **(a)** The generalized formula of Balmer

$$\lambda = \frac{91.18 \text{ m}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

with $m = 3$, and $n > 3$ accounts for a series of spectral lines. This series is called the Paschen series and the wavelengths are

$$\lambda = \frac{91.18 \text{ nm}}{\left(\frac{1}{3^2} - \frac{1}{n^2}\right)} = \frac{820.62\, n^2}{n^2 - 9}$$

The first four members are $\lambda_1 = 1876$ nm, $\lambda_2 = 1282$ nm, $\lambda_3 = 1094$ nm, and $\lambda_4 = 1005$ nm

(b) The Paschen series converges when $n \to \infty$. This means

$$\frac{1}{n^2} \to 0 \Rightarrow \lambda \to \frac{91.18 \text{ nm}}{\left(\frac{1}{3}\right)^2} = 820.6 \text{ nm}$$

(c) For a diffraction grating, the condition for bright (constructive interference) fringes is $d\sin\theta_p = p\lambda,$ where $p =$ 1, 2, 3, … For first-order diffraction, this equation simplifies to $d\sin\theta = \lambda$. For the first and second members of the Paschen series, the condition is $d\sin\theta_1 = \lambda_1$ and $d\sin\theta_2 = \lambda_2$. Dividing these two equations yields

$$\sin\theta_2 = \sin\theta_1\left(\frac{\lambda_2}{\lambda_1}\right) = \sin\theta_1\left(\frac{1282 \text{ nm}}{1876 \text{ nm}}\right) = (0.6834)\sin\theta_1$$

The distance from the center to the first maximum is $y = L\tan\theta$. Thus,

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.607 \text{ m}}{1.5 \text{ m}} = 0.4047 \Rightarrow \theta_1 = 22.03° \Rightarrow \sin\theta_2 = (0.6834)\sin 22.03° \Rightarrow \theta_2 = 14.85°$$

Applying the position formula once again,

$y_2 = L\tan\theta_2 = (1.5 \text{ m})\tan 14.85° = 0.398 \text{ m} = 39.8 \text{ cm}$

25.21. Model: Use the photon model of light.

Solve: **(a)** The wavelength is calculated as follows:

$$E_{\text{gamma}} = hf = h\left(\frac{c}{\lambda}\right) \Rightarrow \lambda = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)\left(3.0\times10^{8}\ \text{m/s}\right)}{1.0\times10^{-13}\ \text{J}} = 2.0\times10^{-12}\ \text{m}$$

(b) The energy of a visible-light photon of wavelength 500 nm is

$$E_{\text{visible}} = h\left(\frac{c}{\lambda}\right) = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)\left(3.0\times10^{8}\ \text{m/s}\right)}{500\times10^{-9}\ \text{m}} = 3.978\times10^{-19}\ \text{J}$$

The number of photons *n* such that $E_{\text{gamma}} = nE_{\text{visible}}$ is

$$n = \frac{E_{\text{gamma}}}{E_{\text{visible}}} = \frac{1.0\times10^{-13}\ \text{J}}{3.978\times10^{-19}\ \text{J}} = 2.5\times10^{5}$$

25.22. Model: Use the photon model.

Solve: The energy of a 1000 kHz photon is

$$E_{\text{photon}} = hf = \left(6.63\times10^{-34}\ \text{Js}\right)\left(1000\times10^{3}\ \text{Hz}\right) = 6.63\times10^{-28}\ \text{J}$$

The energy transmitted each second is 20×10^{3} J. The number of photons transmitted each second is $20\times10^{3}\ \text{J}/6.63\times10^{-28}\ \text{J} = 3.0\times10^{31}$.

25.23. Model: Use the photon model for the laser light.

Solve: **(a)** The energy is

$$E_{\text{photon}} = hf = h\left(\frac{c}{\lambda}\right) = \left(6.63\times10^{-34}\ \text{Js}\right)\left(\frac{3\times10^{8}\ \text{m/s}}{633\times10^{-9}\ \text{m}}\right) = 3.1\times10^{-19}\ \text{J}$$

(b) The energy emitted each second is 1.0×10^{-3} J. The number of photons emitted each second is $1.0\times10^{-3}\ \text{J}/3.14\times10^{-19}\ \text{J} = 3.2\times10^{15}$.

25.24. Model: Use the photon model for the incandescent light.

Solve: The photons travel in all directions. At a distance of r from the light bulb, the photons spread over a sphere of surface area $4\pi r^2$. The number of photons per second per unit area at the location of your retina is

$$\frac{3\times10^{18}\ \text{s}^{-1}}{4\pi\left(10\times10^{3}\ \text{m}\right)^2}=2.387\times10^{9}\ \text{s}^{-1}\ \text{m}^{-2}$$

The number of photons that enter your pupil per second is

$$2.387\times10^{9}\ \text{s}^{-1}\text{m}^{-2}\times\pi\left(3.5\times10^{-3}\ \text{m}\right)^2=9.2\times10^{4}\ \text{s}^{-1}$$

25.25. Model: Use the photon model of light and the Bragg condition for diffraction.

Solve: The Bragg condition for the reflection of x-rays from a crystal is $2d\cos\theta_m = m\lambda$. To determine the angles of incidence θ_m, we need to first calculate λ. The wavelength is related to the photon's energy as $E = hc/\lambda$. Thus,

$$\lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)\left(3.0\times10^{8}\ \text{m/s}\right)}{1.50\times10^{-15}\ \text{J}} = 1.326\times10^{-10}\ \text{m}$$

From the Bragg condition,

$$\theta_m = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left[\frac{\left(1.326\times10^{-10}\ \text{m}\right)m}{2\left(0.21\times10^{-9}\ \text{m}\right)}\right] = \cos^{-1}\left(0.3157m\right) \Rightarrow \theta_1 = \cos^{-1}\left(0.3157\right) = 71.6°$$

Likewise, $\theta_2 = \cos^{-1}\left(0.3157\times2\right) = 50.8°$ and $\theta_3 = 18.7°$. Note that $\theta_4 = \cos^{-1}\left(0.3157\times4\right)$ is not allowed because the $\cos\theta$ cannot be larger than 1. Thus, the x-rays will be diffracted at angles of incidence equal to 18.7°, 50.8°, and 71.6°.

25.26. Model: The angles for which diffraction from parallel planes occurs satisfy the Bragg condition.
Solve: We cannot assume that these are the first and second order diffractions. The Bragg condition is $2d\cos\theta_m = m\lambda$. We have

$$2d\cos 45.6° = m\lambda \qquad 2d\cos 21.0° = (m+1)\lambda$$

Notice that θ_m decreases as m increases, so 21.6° corresponds to the larger value of m. Dividing these two equations,

$$\frac{\cos 45.6°}{\cos 21.0°} = \frac{m}{m+1} = 0.7494 \Rightarrow m = 3$$

Thus these are the third and fourth order diffractions. Substituting into the Bragg condition,

$$d = \frac{3\times 0.0700\times 10^{-9}\ \text{m}}{2\cos 45.6°} = 1.50\times 10^{-10}\ \text{m} = 0.150\ \text{nm}$$

25.27. Model: The x-ray diffraction angles satisfy the Bragg condition.

Solve: **(a)** The Bragg condition ($2d\cos\theta_m = m\lambda$) for normal incidence, $\theta_m = 0°$, simplifies to $2d = m\lambda$. For a thin film of a material on a substrate where $n_{air} < n_{material} < n_{substrate}$, constructive interference between the two reflected waves occurs when $2d = m\lambda$, where λ is the wavelength inside the material.

(b) From a thin film with a period of 1.2 nm, that is, with d = 1.2 nm, the two longest x-ray wavelengths that will reflect at normal incidence are

$$\lambda_1 = \frac{2d}{1} \qquad \lambda_2 = \frac{2d}{2}$$

This means that $\lambda_1 = 2(1.2 \text{ nm}) = 2.4 \text{ nm}$ and $\lambda_2 = 1.2 \text{ nm}$.

25.28. Solve: A small fraction of the light wave of an appropriate wavelength is reflected from each little "bump" in the refractive index. These little bumps act like the atomic planes in a crystal. The light will be strongly reflected (and hence blocked in transmission) if it satisfies the Bragg condition at normal incidence ($\theta = 0$).

$$2d = m\lambda_{\text{glass}} = \frac{m\lambda}{n_{\text{glass}}} \Rightarrow \lambda = \frac{2dn_{\text{glass}}}{m} = \frac{2\left(0.45\times10^{-6}\text{ m}\right)\left(1.50\right)}{1} = 1.35\ \mu\text{m}$$

25.29. Model: Particles have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the particles causes an interference pattern in a double-slit apparatus.

Solve: **(a)** Since the speed of the neutron and electron are the same, the neutron's momentum is

$$p_{\mathrm{n}} = m_{\mathrm{n}} v_{\mathrm{n}} = \frac{m_{\mathrm{n}}}{m_{\mathrm{e}}} m_{\mathrm{e}} v_{\mathrm{n}} = \frac{m_{\mathrm{n}}}{m_{\mathrm{e}}} m_{\mathrm{e}} v_{\mathrm{e}} = \frac{m_{\mathrm{n}}}{m_{\mathrm{e}}} p_{\mathrm{e}}$$

where m_{n} and m_{e} are the neutron's and electron's masses. The de Broglie wavelength for the neutron is

$$\lambda_{\mathrm{n}} = \frac{h}{p_{\mathrm{n}}} = \frac{h}{p_{\mathrm{e}}} \frac{p_{\mathrm{e}}}{p_{\mathrm{n}}} = \lambda_{\mathrm{e}} \frac{m_{\mathrm{e}}}{m_{\mathrm{n}}}$$

From Section 22.2 on double-slit interference, the fringe spacing is $\Delta y = \lambda L/d$. Thus, the fringe spacing for the electron and neutron are related by

$$\Delta y_{\mathrm{n}} = \frac{\lambda_{\mathrm{n}}}{\lambda_{\mathrm{e}}} \Delta y_{\mathrm{e}} = \frac{m_{\mathrm{e}}}{m_{\mathrm{n}}} \Delta y_{\mathrm{e}} = \left(\frac{9.11\times10^{-31}\ \mathrm{kg}}{1.67\times10^{-27}\ \mathrm{kg}} \right) \left(1.5\times10^{-3}\ \mathrm{m}\right) = 8.18\times10^{-7}\ \mathrm{m} = 0.818\ \mu\mathrm{m}$$

(b) If the fringe spacing has to be the same for the neutrons and the electrons,

$$\Delta y_{\mathrm{e}} = \Delta y_{\mathrm{n}} \Rightarrow \lambda_{\mathrm{e}} = \lambda_{\mathrm{n}} \Rightarrow \frac{h}{m_{\mathrm{e}} v_{\mathrm{e}}} = \frac{h}{m_{\mathrm{n}} v_{\mathrm{n}}} \Rightarrow v_{\mathrm{n}} = v_{\mathrm{e}} \frac{m_{\mathrm{e}}}{m_{\mathrm{n}}} = \left(2.0\times10^{6}\ \mathrm{m/s}\right) \left(\frac{9.11\times10^{-31}\ \mathrm{kg}}{1.67\times10^{-27}\ \mathrm{kg}} \right) = 1.1\times10^{3}\ \mathrm{m/s}$$

25.30. Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the electrons causes a diffraction pattern.

Solve: The width of the central maximum of a single-slit diffraction pattern is given by Equation 22.22:

$$w = \frac{2\lambda L}{a} = \frac{2Lh}{ap} = \frac{2Lh}{amv} = \frac{2(1.0\text{ m})(6.63\times10^{-34}\text{ Js})}{(1.0\times10^{-6}\text{ m})(9.11\times10^{-31}\text{ kg})(1.5\times10^{6}\text{ m/s})} = 9.7\times10^{-4}\text{ m} = 0.97\text{ mm}$$

25.31. Model: Neutrons have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the neutrons causes a double-slit interference pattern.

Solve: Measurements show that the spacing between the m = 1 and m = –1 peaks is 1.4 times as long as the length of the reference bar, which gives the real fringe separation Δy = 70 μm. Similarly, the spacing between the m = 2 and m = –2 is 2.8 times as long as the length of the reference bar and yields Δy = 70 μm.

The fringe separation in a double-slit experiment is $\Delta y = \lambda L/d$. Hence,

$$\lambda = \frac{\Delta y\, d}{L} \Rightarrow \frac{h}{p} = \frac{h}{mv} = \frac{\Delta y\, d}{L} \Rightarrow v = \frac{hL}{\Delta y\, md} = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)\left(3.0\ \text{m}\right)}{\left(70\times10^{-6}\ \text{m}\right)\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.10\times10^{-3}\ \text{m}\right)} = 170\ \text{m/s}$$

25.32. Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$.

Visualize: Please refer to Figure 25.11. Notice that a scattering angle $\phi = 60°$ corresponds to an angle of incidence $\theta = 30°$.

Solve: Equation 25.6 describes the Davisson-Germer experiment: $D\sin(2\theta_m) = m\lambda$. Assuming m = 1, this equation simplifies to $D\sin 2\theta = \lambda$. Using $\lambda = h/mv$, we have

$$D = \frac{h}{mv\sin 2\theta} = \frac{6.63\times10^{-34}\ \text{Js}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(4.30\times10^{6}\ \text{m/s}\right)\sin\left(60°\right)} = 1.95\times10^{-10}\ \text{m} = 0.195\ \text{nm}$$

25.33. Model: A confined particle can have only discrete values of energy.
Solve: **(a)** Equation 25.14 simplifies to

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.70\times10^{-9}\ \text{m}\right)^2} = \left(1.231\times10^{-19}\ \text{J}\right)n^2$$

Thus, $E_1 = \left(1.231\times10^{-19}\,\text{J}\right)\left(1^2\right) = 1.2\times10^{-19}\,\text{J}$, $E_2 = \left(1.231\times10^{-19}\,\text{J}\right)\left(2^2\right) = 4.9\times10^{-19}\,\text{J}$, and $E_3 = 1.1\times10^{-18}\,\text{J}$.

(b) The energy is $E_2 - E_1 = 4.9\times10^{-19}\ \text{J} - 1.2\times10^{-19}\ \text{J} = 3.7\times10^{-19}\ \text{J}$.

(c) Because energy is conserved, the photon will carry an energy of $E_2 - E_1 = 3.69\times10^{-19}\ \text{J}$. That is,

$$E_2 - E_1 = E_{\text{photon}} = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_2 - E_1} = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)\left(3.0\times10^8\ \text{m/s}\right)}{3.69\times10^{-19}\ \text{J}} = 540\ \text{nm}$$

25.34. Model: A particle confined in a one-dimensional box has discrete energy levels.

Solve: (a) Equation 24.14 for the $n = 1$ state is

$$E_n = \frac{h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)^2}{8\left(10\times10^{-3}\ \text{kg}\right)\left(0.10\ \text{m}\right)^2} = 5.5\times10^{-64}\ \text{J}$$

The minimum energy of the Ping-Pong ball is $E_1 = 5.5\times10^{-64}$ J.

(b) The speed is calculated as follows:

$$E_1 = 5.50\times10^{-64}\,\text{J} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(10\times10^{-3}\ \text{kg}\right)v^2 \Rightarrow v = \sqrt{\frac{2\left(5.50\times10^{-64}\ \text{J}\right)}{10\times10^{-3}\ \text{kg}}} = 3.3\times10^{-31}\ \text{m/s}$$

25.35. Model: A particle confined in a one-dimensional box has discrete energy levels.

Solve: Using Equation 24.14 for $n = 1$ and 2,

$$E_2 - E_1 = \frac{h^2}{8mL^2}\left(2^2 - 1^2\right) \Rightarrow 1.0\times10^{-19}\ \text{J} = \frac{\left(6.63\times10^{-34}\ \text{Js}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)L^2}(3) = \frac{1.809\times10^{-37}\ \text{Jm}^2}{L^2}$$

$$\Rightarrow L = \sqrt{\frac{1.809\times10^{-37}\ \text{Jm}^2}{1.0\times10^{-19}\ \text{J}}} = 1.3\times10^{-9}\ \text{m} = 1.3\ \text{nm}$$

25.36. Visualize: From the figure we see that the wavelength is 2.0 nm. We'll employ Equations 25.8 $(\lambda = h/p)$ and 25.9 $(E = p^2/2m)$ to express the kinetic energy in terms of wavelength.

Solve:

$$E = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}/2.0\ \mathrm{nm}\right)^2}{2\left(9.11\times10^{-31}\,\mathrm{Kg}\right)} = 6.0\times10^{-20}\,\mathrm{J}$$

Assess: This energy is a little less than one eV, which is reasonable.

25.37. Visualize: The strategy is to take ratios to find n and then plug it back in to find L.
Solve:

$$\frac{E_{n+1}}{E_n} = \frac{\frac{h^2}{8mL^2}(n+1)^2}{\frac{h^2}{8mL^2}(n)^2} = \frac{(n+1)^2}{(n)^2} = \frac{6.4\times10^{-13}\,\text{J}}{3.6\times10^{-13}\,\text{J}}$$

Cancel $\times10^{-13}\,\text{J}$ and take square roots.

$$\frac{n+1}{n} = \sqrt{\frac{6.4}{3.6}} = \frac{4}{3} \qquad \Rightarrow \qquad n = 3$$

So $E_3 = 3.6\times10^{-13}\,\text{J}$. Now solve for L.

$$L = \sqrt{\frac{h^2n^2}{8mE_n}} = \frac{hn}{\sqrt{8mE_n}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(3)}{\sqrt{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.6\times10^{-13}\ \text{J}\right)}} = 29\ \text{fm}$$

Assess: This is not an atomic-sized box, but a nuclear-sized box; that's OK for neutrons.

25.38. Model: The allowed energies of a particle of mass m in a two-dimensional square box of side L are

$$E_{nm} = \frac{h^2}{8mL^2}\left(n^2 + m^2\right)$$

Solve: (a) The minimum energy for a particle is for $n = m = 1$:

$$E_{\text{min}} = E_{11} = \frac{h^2}{8mL^2}\left(1^2 + 1^2\right) = \frac{h^2}{4mL^2}$$

(b) The five lowest allowed energies are E_{min}, $\frac{5}{2}E_{\text{min}}$ (for n = 1, m = 2 and n = 2, m = 1), $4E_{\text{min}}$ (for n = 2, m = 2), $5E_{\text{min}}$ (for n = 1, m = 3 and n = 3, m = 1), and $\frac{13}{2}E_{\text{min}}$ (for n = 2, m = 3 and n = 3, m = 2).

25.39. Model: A particle confined in a one-dimensional box of length L has the discrete energy levels given by Equation 24.14.

Solve: (a) Since the energy is entirely kinetic energy,

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{p^2}{2m} = \tfrac{1}{2}mv_n^2 \Rightarrow v_n = \frac{h}{2mL}n \qquad n = 1,\ 2,\ 3,\ \ldots$$

(b) The first allowed velocity is

$$v_1 = \frac{6.63\times10^{-34}\,\text{Js}}{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.20\times10^{-9}\ \text{m}\right)} = 1.82\times10^{6}\ \text{m/s}$$

For $n = 2$ and $n = 3$, $v_2 = 3.64 \times 10^6$ m/s and $v_3 = 5.46 \times 10^6$ m/s.

25.40. Model: Sets of parallel planes in a crystal diffract x-rays.
Visualize: Please refer to Figure CP25.40.
Solve: The Bragg diffraction condition is $2d\cos\theta_m = m\lambda$, where d is the interplanar separation. Because smaller m values correspond to higher angles of incidence, the diffraction angle of 71.3° in the x-ray intensity plot must correspond to $m = 1$. This means

$$2d\cos 71.3° = 1\left(0.10\times10^{-9}\ \text{m}\right) \Rightarrow d = \frac{0.10\times10^{-9}\ \text{m}}{2\left(\cos 71.3°\right)} = 1.56\times10^{-10}\ \text{m}$$

The cosines of the three angles in the x-ray intensity plot are $\cos 71.3° = 0.321$, $\cos 50.1° = 0.642$, and $\cos 15.8° = 0.962$. These are in the ratio $1:2:3$, which tells us that these are the $m = 1$, 2, and 3 diffraction peaks from a single set of planes with $d = 0.156$ nm.

We can see from the figure that the atomic spacing D of this crystal is related to the interplanar separation d by

$$D = \frac{d}{\sin 60°} = \frac{0.156\ \text{nm}}{\sin 60°} = 0.18\ \text{nm}$$

25.41. **Model:** Sets of parallel planes in a crystal diffract x-rays.
Visualize: Please refer to Figure 25.7.
Solve: **(a)** The Bragg diffraction condition is $2d\cos\theta_m = m\lambda$. The plane spacing is $d_A = 0.20$ nm and the x-ray wavelength is $\lambda = 0.12$ nm. Thus

$$\cos\theta_m = \frac{m\lambda}{2d_A} = \frac{m(0.12\times10^{-9}\text{ m})}{2(0.20\times10^{-9}\text{ m})} = (0.3)m \Rightarrow \theta_{A1} = \cos^{-1}(0.3) = 72.5°$$

Likewise for m = 2 and m = 3, $\theta_{A2} = \cos^{-1}(0.6) = 53.1°$ and $\theta_{A3} = \cos^{-1}(0.9) = 25.8°$. These three angles for the x-ray diffraction peaks match the peaks shown in Figure 25.7c.
(b) The new interplaner spacing is $d_B = d_A/\sqrt{2} = 0.141$ nm (see Figure 25.7b). The Bragg condition for the tilted atomic planes becomes

$$\cos\theta_m = \frac{m\lambda}{2d_B} = 0.4243m$$

For $m = 1$, $\theta_{B1} = \cos^{-1}(0.4243) = 64.9°$. For $m = 2$, $\theta_{B2} = \cos^{-1}(0.8486) = 31.9°$.
(c) The crystal is already tipped by 45° to get the tilted planes (see Figure 25.7b). So, for m = 1, $\theta_1 = 64.9° - 45° =$ 19.9°. $\theta = 64.9° + 45° = 109.9°$ also, but we can't see beyond 90°. For m = 2, $\theta_2 = 31.9° + 45° = 76.9°$. These two angles match the angles in the diffraction peaks of the tilted planes.

25.42. Model: This is an integrated problem that uses concepts from Chapter 22. There are two L's in the problem: L in Chapter 22 refers to the screen distance from the slits, and the L we want here is the length of the box. The wavelength of the neutron determined by the two-slit pattern is the same as the wavelength in the confined box.

Visualize: The figure shows $L_{\text{box}} = 2\lambda$.

We also need Equation 22.6: $y_m = \dfrac{m\lambda L_{\text{screen}}}{d}$. Also from the figure we see that $y_2 = 0.20\times10^{-3}\,\text{m}$. We are given $L_{\text{screen}} = 2.0\text{ m}$ and $d = 15\times10^{-6}\,\text{m}$.

Solve: Solve Equation 22.6 for λ.

$$\lambda = \frac{dy_m}{mL_{\text{screen}}}$$

$$L_{\text{box}} = 2\lambda = 2\frac{dy_m}{mL_{\text{screen}}} = 2\frac{(15\times10^{-6}\text{ m})(0.20\times10^{-3}\text{ m})}{(2)(2.0\text{ m})} = 1.5\text{ nm}$$

Assess: The two pieces of this problem fit together and make sense together.

25.43. Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$. Trapped electrons in the confinement layer behave like a de Broglie wave in a closed-closed tube or like a string fixed at both ends.

Solve: **(a)** The four longest standing-wave wavelengths in the layer are $\lambda = 2L$, L, $\frac{2}{3}L$, and $\frac{1}{2}L$. This follows from the general relation for closed-closed tubes: $\lambda = 2L/n$. Thus, λ = 10.0 nm, 5.00 nm, 3.33 nm, and 2.50 nm.

(b) We have

$$p = mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{Js}}{\left(9.11\times10^{-31}\ \text{kg}\right)\lambda} = \frac{0.7278\times10^{-3}\ \text{m}^2/\text{s}}{\lambda}$$

Using the above four longest values of λ we get the four smallest values of v. Thus,

$$v_1 = \frac{0.7278\times10^{-3}\ \text{m}^2/\text{s}}{10.0\times10^{-9}\ \text{m}} = 7.28\times10^{4}\ \text{m/s}$$

$v_2 = 1.46\times10^{5}\ \text{m/s}$, $v_3 = 2.18\times10^{5}\ \text{m/s}$, and $v_4 = 2.91\times10^{5}\ \text{m/s}$.

25.44. Model: As light is diffracted by matter, matter can also be diffracted by light.
Solve: The de Broglie wavelength of the sodium atoms is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{Js}}{\left(3.84\times10^{-26}\ \text{kg}\right)\left(50\ \text{m/s}\right)} = 3.45\times10^{-10}\ \text{m}$$

The slit spacing of the "diffraction grating" is $d = \frac{1}{2}\lambda_{\text{laser}} = \frac{1}{2}600\ \text{nm} = 300\ \text{nm}.$ Using the diffraction grating equation with $m = 1$, we have

$$d\sin\theta = (1)\lambda \Rightarrow \sin\theta = \frac{\lambda}{d} = 1.151\times10^{-3} \approx 1.2\times10^{-3}$$

In the small-angle approximation, $\sin\theta \cong \tan\theta = y/L$. We get

$$y = L\sin\theta = (1.0\ \text{m})\left(1.151\times10^{-3}\right) = 1.2\ \text{mm}$$