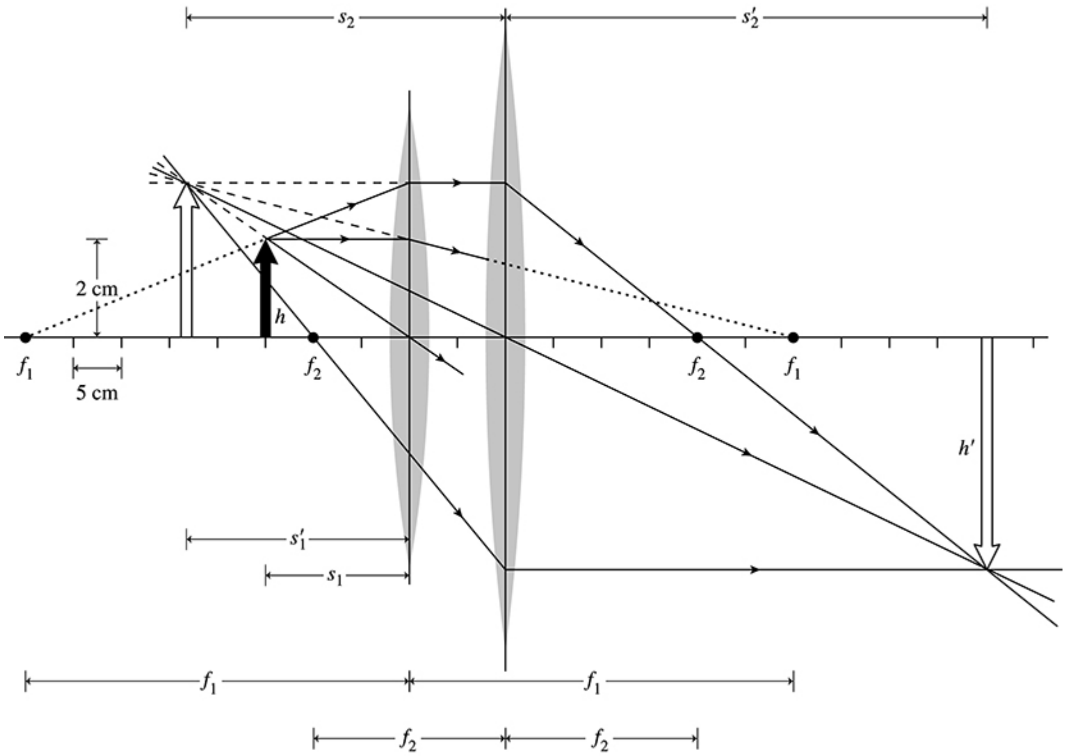
24.1. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray-tracing shows that the lens combination will produce a real, inverted image behind the second lens.

Solve: (a) From the ray-tracing diagram, we find that the image is ≈ 50 cm from the second lens and the height of the final image is 4.5 cm.

(b) s_1 = 15 cm is the object distance of the first lens. Its image, which is a virtual image, is found from the thin-lens equation:

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{40 \text{ cm}} - \frac{1}{15 \text{ cm}} = -\frac{5}{120 \text{ cm}} \Rightarrow s_1' = -24 \text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{(-24 \text{ cm})}{15 \text{ cm}} = 1.6$$

The image of the first lens is now the object for the second lens. The object distance is s_2 = 24 cm + 10 cm = 34 cm. A second application of the thin-lens equation yields:

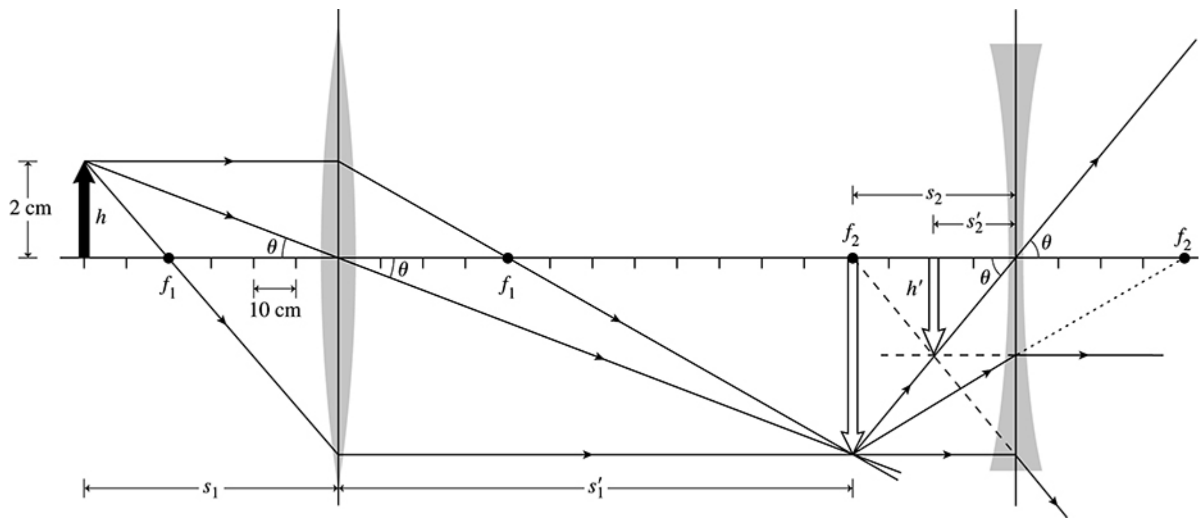
$$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{20 \text{ cm}} - \frac{1}{34 \text{ cm}} \Rightarrow s_2' = \frac{680 \text{ cm}}{14} = 48.6 \text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{48.6 \text{ cm}}{34 \text{ cm}} = -1.429$$

The combined magnification is $m = m_1 m_2 = (1.6)(-1.429) = -2.286$. The height of the final image is (2.286)(2.0 cm) = 4.57 cm. These calculated values are in agreement with those found in part (a).

24.2. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image in front of the second lens.

Solve: (a) From the ray-tracing diagram, we find that the image is 20 cm in front of the second lens and the height of the final image is 2.0 cm.

(b) s_1 = 60 cm is the object distance of the first lens. Its image, which is a real image, is found from the thin-lens equation:

$$\frac{1}{s_1'} = \frac{1}{f_s} - \frac{1}{s_1} = \frac{1}{40 \text{ cm}} - \frac{1}{60 \text{ cm}} = \frac{1}{120 \text{ cm}} \Rightarrow s_1' = 120 \text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{120 \text{ cm}}{60 \text{ cm}} = -2$$

The image of the first lens is now the object for the second lens. The object distance is s_2 = 160 cm −120 cm = 40 cm. A second application of the thin-lens equation yields:

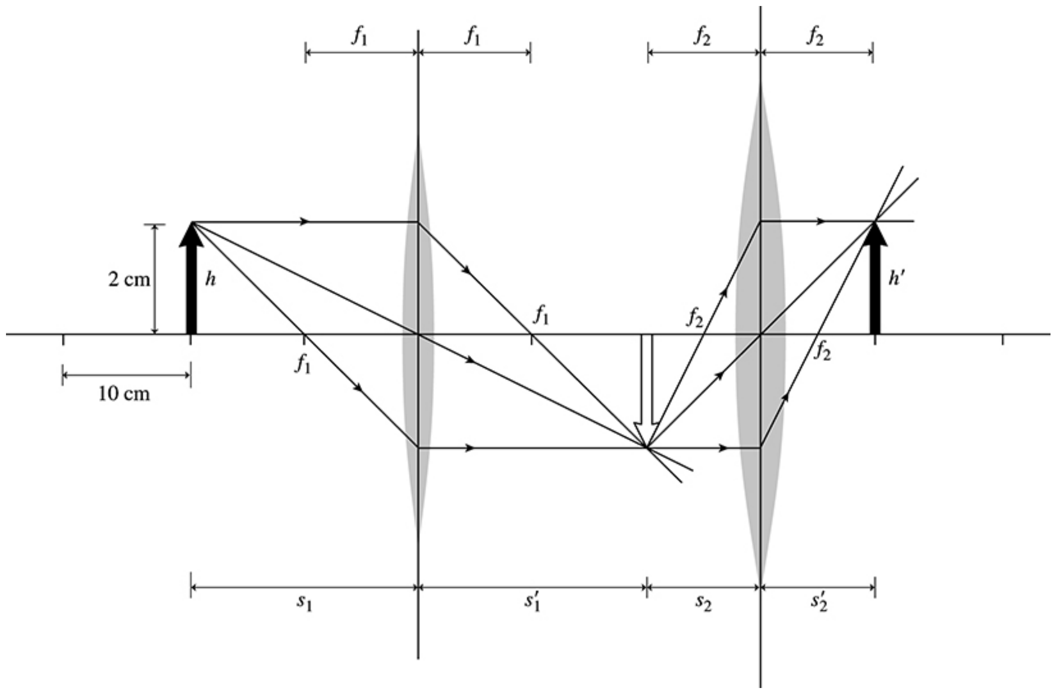
$$\frac{1}{s_2'} = -\frac{1}{s_2} + \frac{1}{f_2} = \frac{-1}{+40 \text{ cm}} + \frac{1}{-40 \text{ cm}} \Rightarrow s_2' = -20 \text{ cm}$$

The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{-20 \text{ cm}}{40 \text{ cm}} = +0.5$$

The overall magnification is $m = m_1 m_2 = (-2)(0.5) = -1.0$. The height of the final image is (+1.0)(2.0 cm) = 2.0 cm. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part (a).

24.3. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a real, upright image behind the second lens.

Solve: (a) From the ray-tracing diagram, we find that the image is 10 cm behind the second lens and the height of the final image is 2 cm.

(b) $s_1 = 20$ cm is the object distance of the first lens. Its image, which is real and inverted, is found from the thin lens equation:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow s_1'=\frac{f_1 s_1}{s_1-f_1}=\frac{(10\text{ cm})(20\text{ cm})}{20\text{ cm}-10\text{ cm}}=20\text{ cm}$$

The magnification of the first lens is

$$m_1=-\frac{s_1'}{s_1}=-\frac{20\text{ cm}}{20\text{ cm}}=-1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30\text{ cm}-20\text{ cm}=10\text{ cm}.$ A second application of the thin-lens equation yields

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow s_2'=\frac{f_2 s_2}{s_2-f_2}=\frac{(5\text{ cm})(10\text{ cm})}{10\text{ cm}-5\text{ cm}}=10\text{ cm}$$

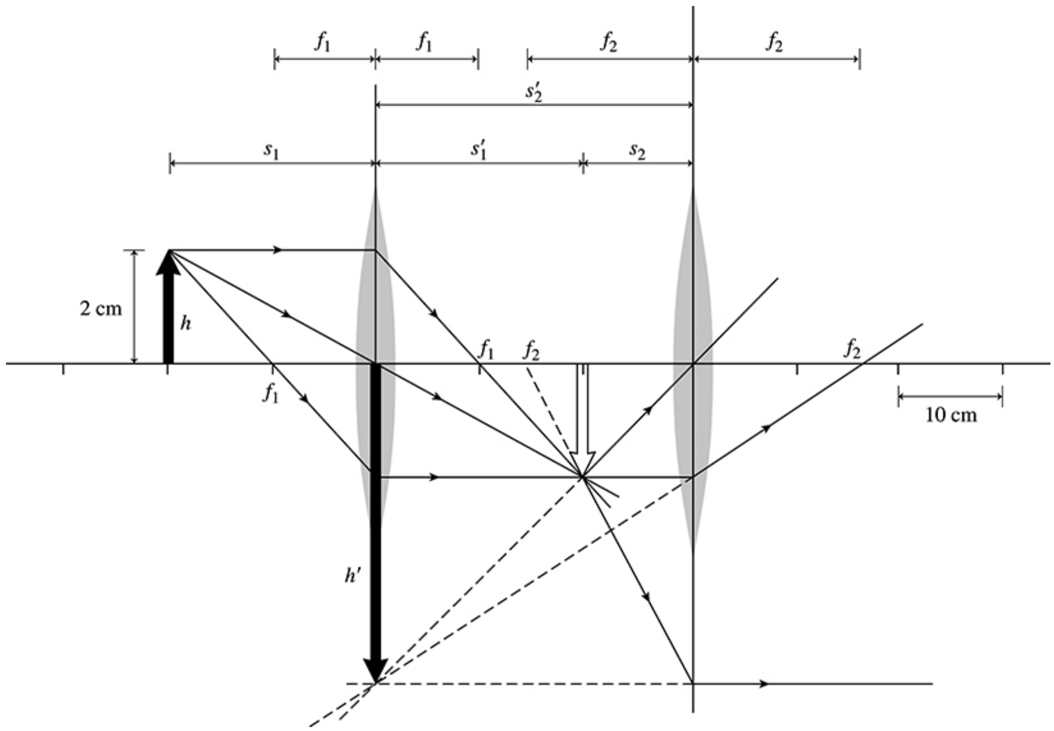
The magnification of the second lens is

$$m_2=-\frac{s_2'}{s_2}=-\frac{10\text{ cm}}{10\text{ cm}}=-1$$

The combined magnification is $m=m_1m_2=(-1)(-1)=1.$ The height of the final image is $(1)(2.0\text{ cm})=2.0\text{ cm}.$ These calculated values are in agreement with those found in part (a).

Assess: The thin-lens equation agrees with the ray tracing.

24.4. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image at the first lens.

Solve: **(a)** From the ray-tracing diagram, we find that the image is 30 cm in front of the second lens and the height of the final image is 6 cm.

(b) $s_1 = 20 \text{ cm}$ is the object distance of the first lens. Its image, which is real and inverted, is found from the thin lens equation:

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1} \Rightarrow s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10 \text{ cm})(20 \text{ cm})}{20 \text{ cm} - 10 \text{ cm}} = 20 \text{ cm}$$

The magnification of the first lens is

$$m_1 = -\frac{s_1'}{s_1} = -\frac{20 \text{ cm}}{20 \text{ cm}} = -1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30 \text{ cm} - 20 \text{ cm} = 10 \text{ cm}.$ A second application of the thin lens equation yields

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(15 \text{ cm})(10 \text{ cm})}{10 \text{ cm} - 15 \text{ cm}} = -30 \text{ cm}$$

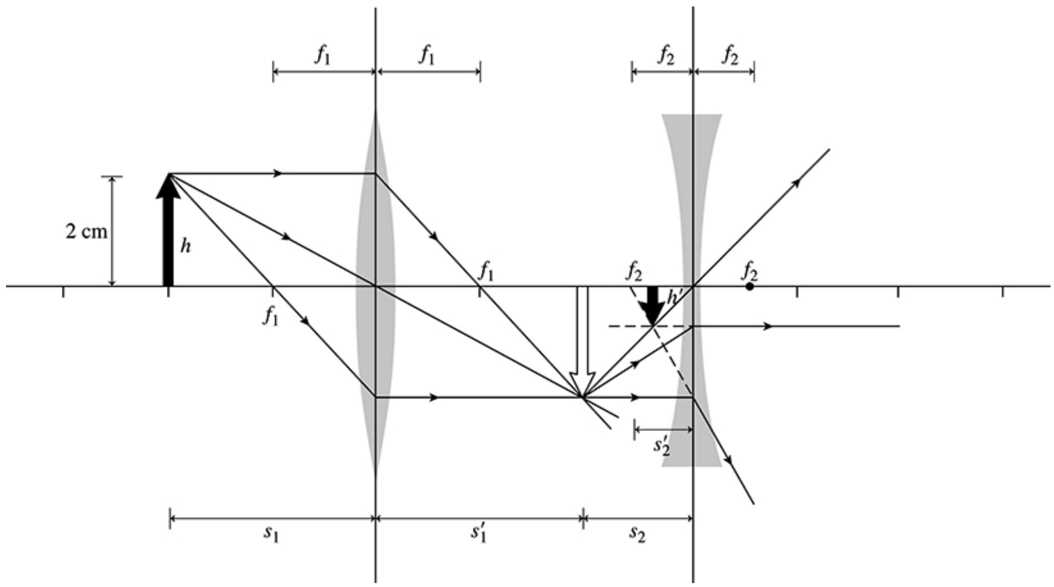
The magnification of the second lens is

$$m_2 = -\frac{s_2'}{s_2} = -\frac{(-30 \text{ cm})}{10 \text{ cm}} = 3$$

The combined magnification is $m = m_1 m_2 = (-1)(3) = -3.$ The height of the final image is $(3)(2.0 \text{ cm}) = 6.0 \text{ cm}.$ The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part (a).

Assess: The thin lens equation agrees with the ray tracing.

24.5. Model: Each lens is a thin lens. The image of the first lens is the object for the second lens.
Visualize:



The figure shows the two lenses and a ray-tracing diagram. The ray tracing shows that the lens combination will produce a virtual, inverted image in front of the second lens.
Solve: **(a)** From the ray-tracing diagram, we find that the image is 3.3 cm behind the second lens and the height of the final image is $\approx 0.7\,\text{cm}$.

(b) $s_1 = 20$ cm is the object distance of the first lens. Its image, which is real and inverted, is found from the thin-lens equation:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow s_1'=\frac{f_1 s_1}{s_1-f_1}=\frac{(10\text{ cm})(20\text{ cm})}{20\text{ cm}-10\text{ cm}}=20\text{ cm}$$

The magnification of the first lens is

$$m_1=-\frac{s_1'}{s_1}=-\frac{20\text{ cm}}{20\text{ cm}}=-1$$

The image of the first lens is now the object for the second lens. The object distance is $s_2 = 30\text{ cm} - 20\text{ cm} = 10\text{ cm}$. A second application of the thin-lens equation yields

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow s_2'=\frac{f_2 s_2}{s_2-f_2}=\frac{(-5\text{ cm})(10\text{ cm})}{10\text{ cm}+5\text{ cm}}=-3.33\text{ cm}$$

The magnification of the second lens is

$$m_2=-\frac{s_2'}{s_2}=-\frac{(-3.3\text{ cm})}{10\text{ cm}}=0.33$$

The combined magnification is $m = m_1 m_2 = (-1)(0.33) = -0.33$. The height of the final image is $(0.33)(2.0\text{ cm}) = 0.66\text{ cm}$. The image is inverted because m has a negative sign. These calculated values are in agreement with those found in part (a).
Assess: The thin-lens equation agrees with the ray tracing.

24.6. Model: $s \gg f$ so we can use Equation 24.1: $m = -f/s$.

Solve:

$$h' = mh = -\frac{f}{s}h = -\frac{15\text{ mm}}{10\text{ m}}(2.0\text{ m}) = -3.0\text{ mm}$$

The height of the image on the detector is 3.0 mm.

Assess: This seems reasonable given typical focal lengths and detector sizes.

24.7. Visualize: Equation 24.2 gives $f\text{-number} = f/D$.

Solve:

$$f\text{-number} = \frac{f}{D} = \frac{35\ \text{mm}}{7.0\ \text{mm}} = 5.0$$

Assess: This is in the range of f-numbers for typical camera lenses.

24.8. Visualize: Solve Equation 24.2 for D.

Solve:

$$D = \frac{f}{f\text{-number}} = \frac{12\ \text{mm}}{4.0} = 3.0\ \text{mm}$$

Assess: This is in the same ballpark as the example after Equation 24.2.

24.9. Visualize: First we compute the f-number of the first lens and then the diameter of the second.

Solve:

$$f\text{-number} = \frac{f}{D} = \frac{12\text{ mm}}{4.0\text{ mm}} = 3.0$$

Now for the new lens.

$$D = \frac{f}{f\text{-number}} = \frac{18\text{ mm}}{3.0} = 6.0\text{ mm}$$

Assess: Given the same f-number, the longer focal length lens has a larger diameter.

24.10. Visualize: We want the same exposure in both cases. The exposure depends on $I\Delta t_{\text{shutter}}$. We'll also use Equation 24.3.

Solve:

$$\text{exposure} = I\Delta t \propto \frac{1}{(f\text{-number})^2}\Delta t$$

$$\frac{1}{(f\text{-number})^2}\Delta t = \frac{1}{(f\text{-number})'^2}\Delta t'$$

$$\Delta t' = \frac{(f\text{-number})'^2}{(f\text{-number})^2}\Delta t = \frac{(4.0)^2}{(5.6)^2}\left(\frac{1}{125}\text{s}\right) = \frac{1}{245}\text{ s} \approx \frac{1}{250}\text{s}$$

Assess: An alternate approach without a lot of calculation is that since we changed the lens (opened) by one *f* stop that doubles the intensity so we need half the time interval to achieve the same exposure.

24.11. Visualize: We want the same exposure in both cases. The exposure depends on $I\Delta t_{\text{shutter}}$. We'll also use Equation 24.3. The lens is the same lens in both cases, so $f = f'$.

Solve:

$$\text{exposure} = I\Delta t \propto \frac{D^2}{f^2}\Delta t$$

$$\frac{D^2}{f^2}\Delta t = \frac{D'^2}{f'^2}\Delta t'$$

Solve for D'; then simplify.

$$D' = \sqrt{D^2\left(\frac{f'}{f}\right)^2\left(\frac{\Delta t}{\Delta t'}\right)} = D\sqrt{\frac{\Delta t}{\Delta t'}} = (3.0\text{ mm})\sqrt{\frac{1/125\text{s}}{1/500\text{s}}} = (3.0\text{ mm})\sqrt{4} = 6.0\text{ mm}$$

Assess: Since we decreased the shutter speed by a factor of 4 we need to increase the aperture area by a factor of 4, and this means increase the diameter by a factor of 2.

24.12. Model: Ignore the small space between the lens and the eye.

Visualize: Refer to Example 24.4, but we want to solve for s', the near point.

Solve:

(a) The power of the lens is positive which means the focal length is positive, so Ramon wears converging lenses. This is the remedy for hyperopia.

(b) We want to know where the image should be for an object $s = 25 \text{ cm}$ given $1/f = 2.0 \text{ m}^{-1}$.

$$f = \frac{1}{P} = 0.50 \text{ m}$$

$$s' = \frac{fs}{s - f} = \frac{(0.50 \text{ m})(0.25 \text{ m})}{0.25 \text{ m} + 0.50 \text{ m}} = -0.50 \text{ m}$$

So the near point is 50 cm.

Assess: The negative sign on s' is expected because we need the image to be virtual.

24.13. Model: Ignore the small space between the lens and the eye.

Visualize: Refer to Example 24.5, but we want to solve for s', the far point.

Solve:

(a) The power of the lens is negative which means the focal length is negative, so Ellen wears diverging lenses. This is the remedy for myopia.

(b) We want to know where the image should be for an object $s = \infty$ m given $1/f = -1.0\ \text{m}^{-1}$.

$$f = \frac{1}{P} = -1.0\ \text{m}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

When $s = \infty$ m,

$$\frac{1}{\infty\ \text{m}} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = f = -1.0\ \text{m}$$

So the far point is 100 cm.

Assess: The negative sign on s' is expected because we need the image to be virtual.

24.14. Model: With normal vision the farthest distance at which a relaxed eye can focus (the far point) is infinity. Rays coming from infinity are nearly parallel, so the focal length of the lens would be 24 mm, the same as the length of the eye. However, the far point is less than infinity for many people, and most sources quote the focal length of the eye as 17 mm–22 mm. However, for simplicity of calculation, assume the vision is normal and the focal length of the lens/cornea combination is 24 mm

Visualize: Equation 24.2 gives *f*-number $= f/D$.

Solve: **(a)** For the fully dilated pupil (dark-adapted eye):

$$f\text{-number} = \frac{f}{D} = \frac{24\text{ mm}}{8.0\text{ mm}} = 3.0$$

(b) For the fully contracted pupil (eye in bright light):

$$f\text{-number} = \frac{f}{D} = \frac{24\text{ mm}}{1.5\text{ mm}} = 16$$

Assess: These answers correspond to the values given in the text.

24.15. Model: The angle subtended by the image is $8\times$ the angle subtended by the object.
Visualize: The angle subtended by the object is h/s.
Solve:

$$\theta = (8\times)\frac{h}{s} = (8\times)\frac{14 \text{ cm}}{1800 \text{ cm}} = 0.0622 \text{ rad} = 3.6°$$

Assess: The binoculars do indeed help.

24.16. Visualize: Equation 24.10 relates the variables in question:

$$M = -\frac{L}{f_{obj}}\frac{25\ \text{cm}}{f_{eye}}$$

We are given $M = 500\times$, $L = 20$ cm, and $f_{eye} = 5.0$ cm

Solve: Solve for f_{obj}.

$$f_{obj} = -\frac{L}{M}\frac{25\ \text{cm}}{f_{eye}} = -\frac{20\ \text{cm}}{-500}\frac{25\ \text{cm}}{5.0\ \text{cm}} = 0.20\ \text{cm} = 2.0\ \text{mm}$$

Assess: This is in the same ball park as the case in the book.

24.17. Visualize: Equation 24.10 relates the variables in question:

$$M = -\frac{L}{f_{obj}}\frac{25\text{ cm}}{f_{eye}}$$

We are given $M = 100\times$, $L = 160$ mm, and $f_{obj} = 8.0$ cm.

Solve: Solve for f_{obj}.

$$f_{eye} = -\frac{L}{f_{obj}}\frac{25\text{ cm}}{M} = -\frac{160\text{ mm}}{8.0\text{ mm}}\frac{25\text{ cm}}{(-100)} = 5.0\text{ cm}$$

Assess: This is the same f_{eye} as in the previous exercise.

24.18. Model: Assume the thin-lens equation is valid. For part (b) refer to Equation 24.11 and the definition of α.

Visualize: We are given $f_{obj} = 9.0$ mm and

$$m = -\frac{s'}{s} = -40 \quad \Rightarrow \quad s' = 40s$$

Solve: (**a**) Use the thin-lens equation.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_{obj}}$$

$$\frac{1}{s} + \frac{1}{40s} = \frac{1}{f_{obj}}$$

$$\frac{41}{40s} = \frac{1}{f_{obj}}$$

$$s = \frac{41}{40} f_{obj} = \frac{41}{40}(9.0 \text{ mm}) = 9.2 \text{ mm}$$

(b) We are given $n = 1.00$. Refer to Figure 24.14a to determine α.

$$\alpha = \tan^{-1}(3.0 \text{ mm}/9.0 \text{ mm}) = \tan^{-1}(1/3)$$

$$\text{NA} = n \sin\alpha = (1.00)\sin\left(\tan^{-1}\left(\frac{1}{3}\right)\right) = 0.32$$

Assess: We used $s = f$ in this calculation as suggested in the text. If we had used $s = 9.2$ mm (from part (a)) we would get $\text{NA} = 0.31$, only a little different. These values of NA are typical for a simple microscope.

24.19. Visualize: We are given $\text{NA} = 0.90$, $L = 160\text{ mm}$, $m_{\text{obj}} = -20$, and the book says $n = 1.46$.

Solve: Start with Equation 24.11.

$$\text{NA} = n\sin\alpha$$

$$\alpha = \sin^{-1}\left(\frac{\text{NA}}{n}\right)$$

$$\tan\alpha = \tan\left(\sin^{-1}\left(\frac{\text{NA}}{n}\right)\right)$$

But from Figure 24.14a we also have $\tan\alpha = \left(\frac{1}{2}\right)D/f_{\text{obj}}$, where D is the diameter of the lens. So combine those two expressions for $\tan\alpha$ and solve for D.

$$D = 2f_{obj}\tan\left(\sin^{-1}\left(\frac{\text{NA}}{n}\right)\right)$$

We need the side calculation using Equation 24.9:

$$m_{\text{obj}} = -\frac{L}{f_{\text{obj}}} \Rightarrow f_{\text{obj}} = -\frac{L}{m_{\text{obj}}}$$

Insert this back in the equation for D.

$$D = 2\left(-\frac{L}{m_{\text{obj}}}\right)\tan\left(\sin^{-1}\left(\frac{\text{NA}}{n}\right)\right)$$

$$D = 2\left(-\frac{160\text{ mm}}{-20}\right)\tan\left(\sin^{-1}\left(\frac{0.90}{1.46}\right)\right) = 13\text{ mm}$$

Assess: The answer seems to be in a reasonable range for objective lens diameter.

24.20. Visualize: Figure 24.15 shows from similar triangles that for the eyepiece lens to collect all the light

$$\frac{D_{\text{obj}}}{f_{\text{obj}}} = \frac{D_{\text{eye}}}{f_{\text{eye}}}$$

We also see from Equation 24.12 that $M = -f_{\text{obj}}/f_{\text{eye}}$. We are given $M = -20$ and $D_{\text{obj}} = 12$ cm.

Solve:

$$D_{\text{eye}} = D_{\text{obj}}\frac{f_{\text{eye}}}{f_{\text{obj}}} = \frac{D_{\text{obj}}}{-M} = \frac{12\text{cm}}{20} = 0.60 \text{ cm} = 6.0 \text{ mm}$$

Assess: The answer is almost as wide as a dark-adapted eye.

24.21. Model: Assume the eyepiece is a simple magnifier with $M_{eye} = 25\text{ cm}/f_{eye}$.

Visualize: $f_{eye} = 25\text{ cm}/10 = 2.5\text{ cm}$.

Solve:

(a) The magnification of a telescope is

$$M = -\frac{f_{obj}}{f_{eye}} = \frac{100\text{ cm}}{2.5\text{ cm}} = 40$$

(b)

$$f\text{-number} = \frac{f}{D} = \frac{1.00\text{ m}}{0.20\text{ m}} = 5.0$$

Assess: These results are in reasonable ranges for magnification and *f*-number.

24.22. Model: Diffraction prevents focusing light to an arbitrarily small point. Model the lens of diameter D as an aperture in front of an ideal lens with an 8.0 cm focal length.

Solve: Assuming that the incoming laser beam is parallel, the focal length of the lens should be 8.0 cm. From Equation 24.13, the minimum spot size in the focal plane of this lens is

$$w = \frac{2.44\lambda f}{D} \Rightarrow 10\times10^{-6}\ \text{m} = \frac{2.44\left(633\times10^{-9}\ \text{m}\right)\left(8.0\times10^{-2}\ \text{m}\right)}{D} \Rightarrow D = 0.012\ \text{m} = 1.2\ \text{cm}$$

24.23. Model: Two objects are marginally resolvable if the angular separation between the objects, as seen from the lens, is $\alpha = 1.22\lambda / D$.

Solve: Let Δy be the separation between the two light bulbs, and let L be their distance from a telescope. Thus,

$$\alpha = \frac{\Delta y}{L} = \frac{1.22\lambda}{D} \Rightarrow L = \frac{\Delta y\, D}{1.22\lambda} = \frac{(1.0\text{ m})(4.0\times10^{-2}\text{ m})}{1.22(600\times10^{-9}\text{ m})} = 55\text{ km}$$

24.24. Visualize: Equation 24.15 gives the smallest resolvable distance: $d_{\text{min}} = 0.61\lambda/\text{NA}$. We are given $\lambda = 500\text{ nm}$ and $\text{NA} = 1.0$.

$$d_{\text{min}} = \frac{0.61\lambda}{\text{NA}} = \frac{(0.61)(500\text{ nm})}{1.0} = 305\text{ nm} \approx 310\text{ nm}$$

Assess: The smallest object one can see is on the order of the wavelength.

24.25. Visualize: Equation 24.15 gives the smallest resolvable distance: $d_{\text{min}} = 0.61\lambda/\text{NA}$. We are given $\lambda = 600$ nm and $d_{\text{min}} = 0.75\,\mu\text{m} = 750$ nm.

Solve:

$$\text{NA} = \frac{0.61\lambda}{d_{\text{min}}} = \frac{(0.61)(600\text{ nm})}{750\text{ nm}} = 0.49$$

Assess: This is in the normal range for NA.

24.26. Visualize: We are given $h_1 = 1.0\text{ cm}, s_1 = 4.0\text{ cm}, f_1 = 5.0\text{ cm},$ and $f_2 = -8.0\text{ cm}.$

Solve: First compute the image from the first lens.

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(5.0\text{ cm})(4.0\text{ cm})}{4.0\text{ cm} - 5.0\text{ cm}} = -20\text{ cm}$$

$$h_1' = -h_1\frac{s_1'}{s_1} = -(1.0\text{ cm})\frac{-20\text{ cm}}{4.0\text{ cm}} = 5.0\text{ cm}$$

This is a virtual, upright image 20 cm to the left of the first lens.
The second lens is 12 cm to the right of the first one, so $s_2 = 32\text{ cm}.$

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(-8.0\text{ cm})(32\text{ cm})}{32\text{ cm} - (-8.0\text{ cm})} = -6.4\text{ cm}$$

$$h_2 = h_1' = 5.0\text{ cm}$$

$$h_2' = -h_2\frac{s_2'}{s_2} = -(5.0\text{ cm})\frac{-6.4\text{ cm}}{32\text{ cm}} = 1.0\text{ cm}$$

This is a virtual, upright image 6.4 cm to the left of the second lens (5.6 cm to the right of the first lens). The image is 1.0 cm tall (the same size as the object).

Assess: Ray tracing confirms these results.

24.27. Model: The parallel rays can be considered to come from an object infinitely far away: $s_1 = \infty$. The lens is a diverging lens.

Visualize: If $s_1 = \infty$ the thin lens equation tells us that $s_1' = f_1'$; we are given that $f_1 = -10$ cm. We are also given for the mirror $f_2 = 10$ cm.

Solve: Since $s_1' = -10$ cm the image is virtual 10 cm to the left of the lens. The image from the lens becomes the object for the mirror $\Rightarrow s_2 = 30$ cm; this is three times the mirror's focal length, or $s_2 = 3f_2$.

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{f_2(3f_2)}{3f_2 - f_2} = \frac{3}{2}f_2 = 15 \text{ cm}$$

Therefore the initial parallel rays are brought to a focus 15 cm to the left of the mirror, or 5 cm to the right of the lens.

Assess: The answer is reasonable and can be verified by ray tracing.

24.28. Visualize: The object is within the focal length of the converging lens, so we expect the image to be upright, virtual, and to the left of the lens. The image of the lens becomes the object for the mirror, and we expect the second image to be upright and virtual behind (to the right of) the mirror.

Solve:

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(5\text{ cm})}{5\text{ cm} - 10\text{ cm}} = -10\text{ cm}$$

The image of the lens becomes the object for the mirror $\Rightarrow s_2 = 15\text{ cm},$ and we expect the second image to be upright and virtual behind (to the right of) the mirror.

$$s_1' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(-30\text{ cm})(15\text{ cm})}{15\text{ cm} - (-30\text{ cm})} = -10\text{ cm}$$

$$h' = h\frac{s_1'}{s_1}\frac{s_2'}{s_2} = (1.0\text{ cm})\left(\frac{-10\text{ cm}}{5.0\text{ cm}}\right)\left(\frac{-10\text{ cm}}{15\text{ cm}}\right) = 1.3\text{ cm}$$

The final image is 10 cm to the right of the mirror, or 15 cm to the right of the lens. It is upright with a height of 1.3 cm.

Assess: Ray tracing will verify the answer.

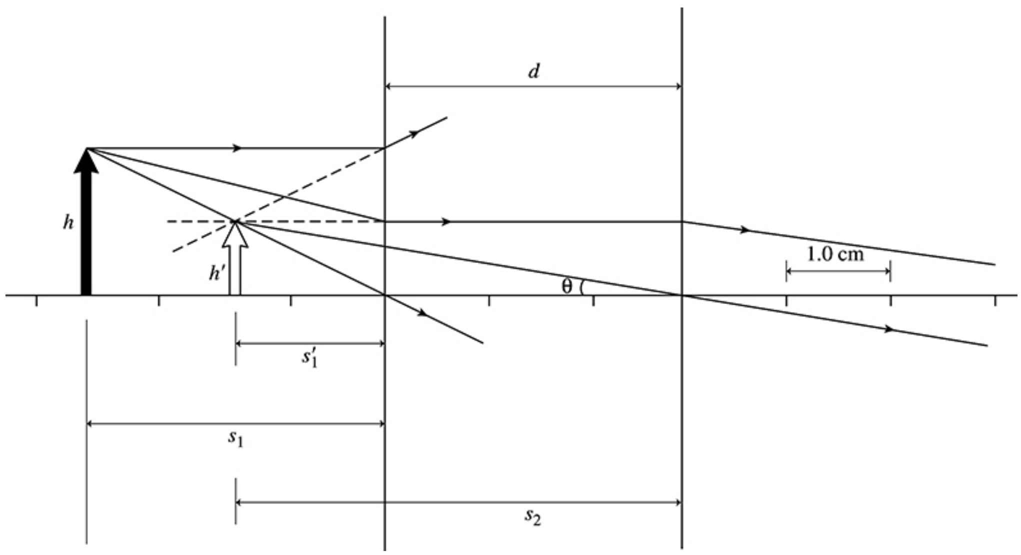
24.29. Solve: **(a)** The image location from the first lens is

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(-2.5\text{ cm})(2.5\text{ cm})}{2.5\text{ cm} - (-2.5\text{ cm})} = -1.25\text{ cm}$$

So the image from the first lens is 1.25 cm to the left of the first lens, upright and virtual.
Now, $s_2 = d + 1.25$ cm.
We are told the final image is at infinity: $s_2' = \infty \Rightarrow s_2 = f_2 \Rightarrow f_2 = d + 1.25$ cm

$$d = f_2 - 1.25\text{cm} = 3.75\text{ cm}$$

(b)



(c)

$$h' = -h\frac{s_1'}{s_1} = 0.50\text{ cm}$$

The angular size is

$$\theta = \tan\frac{h'}{f_2} \approx \frac{h'}{f_2} = \frac{0.50\text{ cm}}{5.0\text{ cm}} = 0.10\text{ rad}$$

(d) If the object were held at the eye's near point, it would subtend:

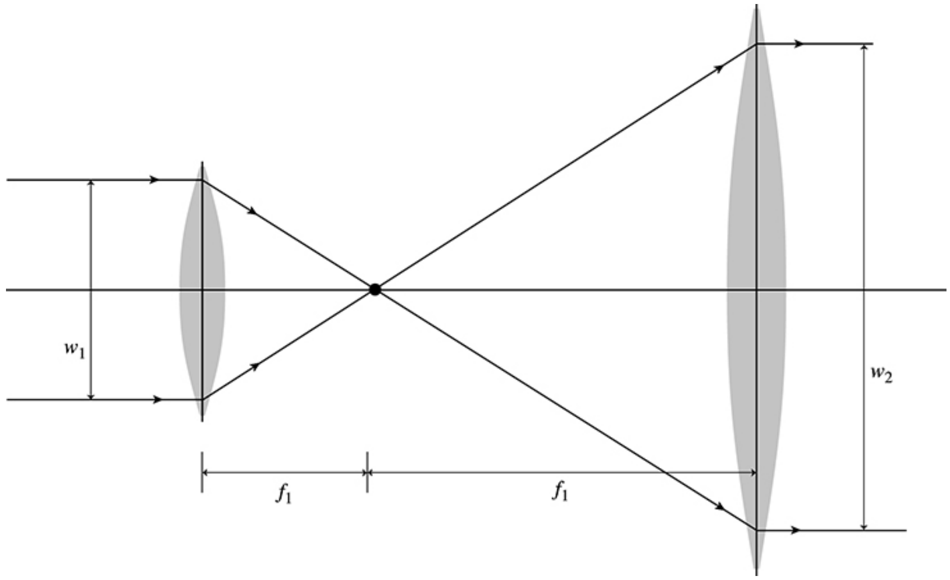
$$\theta_{\text{NP}} = \frac{h}{25\text{ cm}} = \frac{1.0\text{ cm}}{25\text{ cm}} = 0.040\text{ rad}$$

The angular magnification is

$$M = \frac{\theta}{\theta_{\text{NP}}} = \frac{0.10\text{ rad}}{0.040\text{ rad}} = 2.5$$

Assess: The numerical answers seem to agree with the drawing.

24.30. Visualize: See Figure 24.15. Parallel rays coming into the first lens will focus at the focal point of the first lens. If that position is also the focal point of the second lens then the rays will also leave the second lens parallel.



Solve: **(a)** This is similar to a telescope.

$$d = f_1 + f_2$$

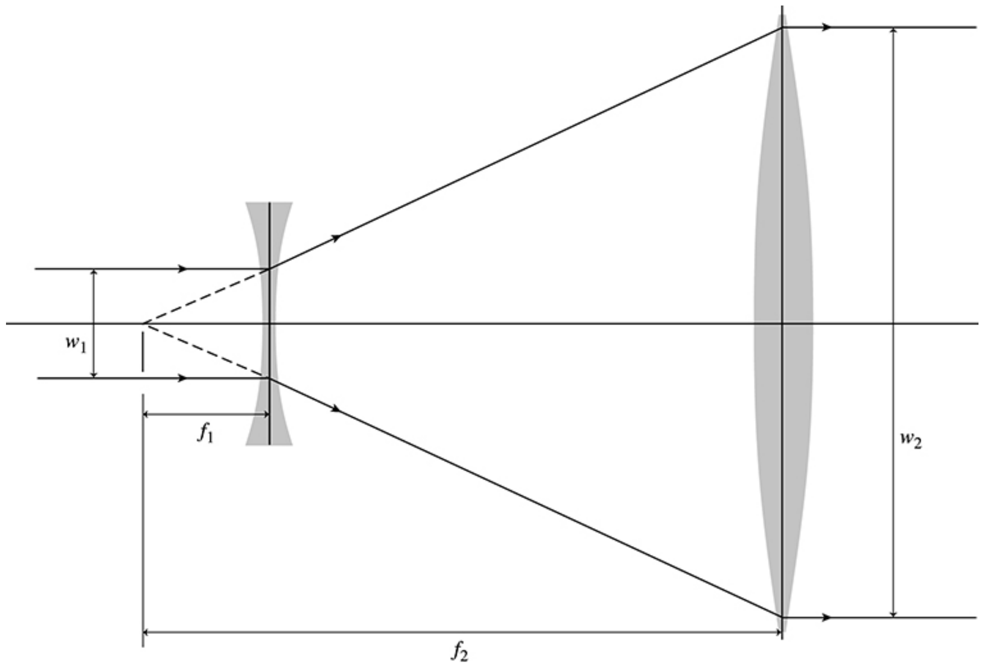
(b) Looking at the similar triangles in the diagram shows that

$$\frac{w_1}{f_1} = \frac{w_2}{f_2}$$

$$w_2 = \frac{f_2}{f_1} w_1$$

Assess: Figure P24.30 says $f_2 > f_1$ and our answer then shows that $w_2 > w_1$ which is the goal of a beam expander.

24.31. Visualize: Hard thought shows that if the left focal points for both lenses coincide then the parallel rays before and after the beam splitter are reproduced. The first lens diverges the rays as if they had come from the focal point of the converging lens.



Solve: **(a)**

$$d = f_2 - |f_1|$$

But since we are given $f_1 < 0,$ this is equivalent to

$$d = f_2 + f_1$$

(b) Looking at the similar triangles in the diagram shows that

$$\frac{w_1}{|f_1|} = \frac{w_2}{f_2}$$

$$w_2 = \frac{f_2}{|f_1|} w_1$$

Assess: Figure P24.31 says $f_2 > |f_1|$ and our answer then shows that $w_2 > w_1$ which is the goal of a beam expander.

24.32. Visualize: We simply need to work backwards. We are given $f_1 = 7.0$ cm and $f_2 = 15$ cm. We are also given $s_2' = -10$ cm. We use this to find s_2.

Solve: (a)

$$s_2 = \frac{f_2 s_2'}{s_2' - f_2} = \frac{(15\text{ cm})(-10\text{ cm})}{-10\text{ cm} - 15\text{ cm}} = 6.0\text{ cm}$$

So the final image is 6.0 cm to the left of the second lens, or 14 cm to the right of the first lens. That is, the object for the second lens is the image from the first lens, so $s_1' = 20\text{ cm} = -6.0\text{ cm} = 14$ cm.

$$s_1 = \frac{f_1 s_1'}{s_1' - f_1} = \frac{(7.0\text{ cm})(14\text{ cm})}{14\text{ cm} - 7.0\text{ cm}} = 14\text{ cm}$$

Thus, $L = 14$ cm.

(b) To find the height and orientation we need to look at the magnification.

$$m = m_1 m_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \left(-\frac{14\text{ cm}}{14\text{ cm}}\right)\left(-\frac{-10\text{ cm}}{6.0\text{ cm}}\right) = -1.7$$

$$h' = hm = (1.0\text{ cm})(-1.7) = -1.7\text{ cm}$$

The negative sign indicates that the image is inverted.

Assess: Ray tracing would verify the answers.

24.33. Model: The plane faces have an infinite radius of curvature. We are assuming the lens(es) are thin.
Visualize: Use Equation 23.27, the lens makers' equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Solve: **(a)** For a symmetric convex lens call $R = |R_1| = |R_2|$ where the sign convention says $R_1 > 0$ and $R_2 < 0$.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{1}{R} + \frac{1}{R}\right) = (n-1)\left(\frac{2}{R}\right)$$

Invert both sides.

$$f = \frac{1}{(n-1)}\frac{R}{2}$$

Now for the plano-convex halves:

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = (n-1)\left(\frac{1}{R} + 0\right) = (n-1)\left(\frac{1}{R}\right)$$

Invert both sides.

$$f_1 = \frac{1}{(n-1)}R$$

$$\frac{1}{f_2} = (n-1)\left(\frac{1}{\infty} - \frac{1}{R_2}\right) = (n-1)\left(0 + \frac{1}{R}\right) = (n-1)\left(\frac{1}{R}\right)$$

Invert both sides.

$$f_2 = \frac{1}{(n-1)}R$$

From the three results we see that $f_1 = f_2 = 2f$.
(b) Use the thin-lens equation.
As a single lens with $s = \frac{1}{2}f$:

$$s' = \frac{fs}{s-f} = \frac{f(\frac{1}{2}f)}{\frac{1}{2}f - f} = -f$$

As a two-lens system with $f_1 = f_2 = 2f$ and $s_1 = \frac{1}{2}f = \frac{1}{4}f_1$:

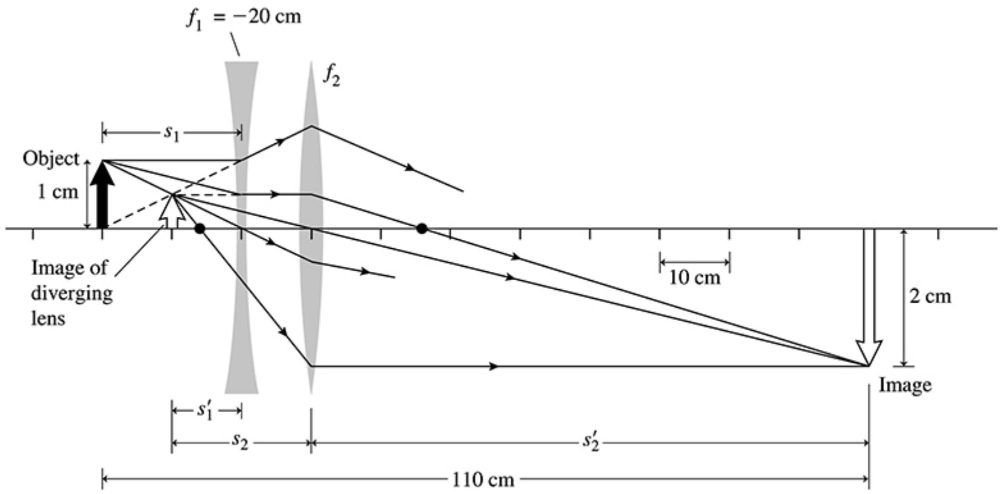
$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(2f)(\frac{1}{2}f)}{\frac{1}{2}f - 2f} = \frac{f^2}{-\frac{3}{2}f} = -\frac{2}{3}f$$

Since this is to the left of the lens $s_2 = \frac{2}{3}f$

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(2f)(\frac{2}{3}f)}{\frac{2}{3}f - 2f} = \frac{\frac{4}{3}f^2}{-\frac{4}{3}f} = -f$$

We get the same result treating it as one symmetric convex lens or as two plano-convex halves (with zero separation).
Assess: We can think of each plano-convex half as providing half the refraction. The answers are consistent.

24.34. **Model:** Use the ray model of light. Assume both the lenses are thin lenses.
Visualize:



Solve: Begin by finding the image of the diverging lens,

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1} \Rightarrow \frac{1}{20\text{ cm}}+\frac{1}{s_1'}=\frac{1}{-20\text{ cm}} \Rightarrow s_1'=-10\text{ cm}$$

This image is the object for the second lens. Its distance from the screen is $s_2+s_2'=110\text{ cm}-10\text{ cm}=100\text{ cm}$. The overall magnification is

$$M=m_1m_2=-\frac{h'}{h}=-2$$

The magnification of the diverging lens is

$$m_1=-\frac{s_1'}{s_1}=-\frac{(-10\text{ cm})}{20\text{ cm}}=\frac{1}{2}$$

Thus the magnification of the converging lens needs to be

$$m_2=-\frac{s_2'}{s_2}=-4 \Rightarrow s_2'=4s_2$$

Substituting this result into $s_2+s_2'=100$ cm, we have $s_2 + 4s_2 = 100$ cm, which means $s_2 = 20$ cm and $s_2'=80$ cm. We can find the focal length by using the thin-lens equation for the converging lens:

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2} \Rightarrow \frac{1}{f_2}=\frac{1}{20\text{ cm}}+\frac{1}{80\text{ cm}} \Rightarrow f_2=16\text{ cm}$$

Hence, the second lens is a converging lens of focal length 16 cm. It must be placed 10 cm in front of the diverging lens, toward the screen, or 80 cm from the screen.

24.35. Model: Yang has myopia. Normal vision will allow Yang to focus on a very distant object. In measuring distances, we'll ignore the small space between the lens and her eye.

Solve: Because Yang can see objects at 150 cm with a fully relaxed eye, we want a lens that creates a virtual image at $s' = -150$ cm (negative because it's a virtual image) of an object at $s = \infty$ cm. From the thin-lens equation,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty \text{ m}} + \frac{1}{-1.5 \text{ m}} = -0.67 \text{ D}$$

So Yang gets a prescription for a -0.67 D lens which has $f = -150$ cm.

Since Yang can accommodate to see things as close as 20 cm we need to create a virtual image at 20 cm of objects that are at $s =$ new near point. That is, we want to solve the thin-lens equation for s when $s' = -20$ cm and $f = -150$ cm.

$$s = \frac{fs'}{s' - f} = \frac{(-150 \text{ cm})(-20 \text{ cm})}{-20 \text{ cm} - (-150 \text{ cm})} = 23 \text{ cm}$$

Assess: Diverging lenses are always used to correct myopia.

24.36. Visualize: Use Equation 23.21:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$$

where $n_1 = 1.00$ for air and $n_2 = 1.34$ for aqueous humor. If we think of incoming parallel rays coming to a focus in the humor then we have $s = \infty$ and $s' = f$.

Solve:

$$\frac{1.0}{\infty}+\frac{n_2}{f}=\frac{n_2-n_1}{R}$$

Solve for R.

$$R = f\frac{n_2-n_1}{n_2}=(3.0\ \text{cm})\frac{1.34-1.00}{1.34}=0.76\ \text{cm}$$

Assess: If you think about the dimensions of an eye, this answer seems physically possible.

24.37. Visualize: Use Equation 23.27, the lens makers' equation:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For a symmetric lens $R_1 = R_2$ and

$$f=\frac{R}{2(n-1)} \quad \text{and} \quad R=2(n-1)f$$

Also needed will be the magnification of a telescope: $M=-f_{\text{obj}}/f_{\text{eye}} \Rightarrow f_{\text{eye}}=-f_{\text{obj}}/M$ (but we will drop the negative sign).

We are given $R_{\text{obj}} = 100$ cm and $M = 20$.

Solve:

$$R_{\text{eye}}=2(n-1)f_{\text{eye}}=2(n-1)\frac{f_{\text{obj}}}{M}=2(n-1)\frac{\frac{R_{\text{obj}}}{2(n-1)}}{M}=\frac{R_{\text{obj}}}{M}=\frac{100\text{ cm}}{20}=5.0\text{ cm}$$

Assess: We expect a short focal length and small radius of curvature for telescope eyepieces.

24.38. Model: Assume that each lens is a simple magnifier with $M = 25\text{ cm}/f$.

Visualize:

$$M_{\text{obj}} = \frac{25\text{ cm}}{f_{\text{obj}}} \Rightarrow f_{\text{obj}} = \frac{25\text{ cm}}{M_{\text{obj}}}$$

$$M_{\text{eye}} = \frac{25\text{ cm}}{f_{eye}} \Rightarrow f_{\text{eye}} = \frac{25\text{ cm}}{M_{\text{eye}}}$$

Solve: **(a)** The magnification of a telescope is

$$M = -\frac{f_{\text{obj}}}{f_{\text{eye}}} = -\frac{\dfrac{25\text{ cm}}{M_{\text{obj}}}}{\dfrac{25\text{ cm}}{M_{\text{eye}}}} = -\frac{M_{\text{eye}}}{M_{\text{obj}}}$$

The way to maximize the magnitude of this is to have $M_{\text{eye}} > M_{\text{obj}}$.

$$M = -\frac{5.0}{2.0} = -2.5$$

The magnification is usually given without the negative sign, so it is $2.5\times$.

(b) To achieve this we used the $2.0\times$ lens as the objective, which coincides with the text which says the objective should have a long focal length and the eyepiece a short focal length.

(c)

$$L = f_{\text{obj}} + f_{\text{eye}} = \frac{25\text{ cm}}{M_{\text{obj}}} + \frac{25\text{ cm}}{M_{\text{eye}}} = \frac{25\text{ cm}}{2.0} + \frac{25\text{ cm}}{5.0} = 17.5\text{ cm}$$

Assess: This is not a very powerful telescope.

24.39. Model: To make a telescope you need an objective with a long focal length and an eyepiece with a short focal length.
Visualize:

$$f = \frac{1}{P}$$

Solve:
(a) The lens with the smaller refractive power has the longer focal length and should be used as the object—that's the lens with $P = +3.0\text{ D}$. The $+4.5\text{ D}$ lens should be used as the eyepiece.
(b)

$$M = -\frac{f_{\text{obj}}}{f_{\text{eye}}} = -\frac{P_{\text{eye}}}{P_{\text{obj}}} = \frac{4.5\text{ D}}{3.0\text{ D}} = -1.5$$

(c) In a telescope the lenses should be a distance apart equal to the sum of their focal lengths.

$$d = f_{\text{obj}} + f_{\text{eye}} = \frac{1}{P_{\text{obj}}} + \frac{1}{P_{\text{eye}}} = \frac{1}{3.0\text{ D}} + \frac{1}{4.5\text{ D}} = 0.56\,\text{m}$$

Assess: These numbers are reasonable, although a $+4.5\text{ D}$ lens is fairly strong. You really could make yourself a telescope by holding the lenses a half-meter apart and get a little $(1.5\times)$ magnification.
This cannot be done with the glasses of nearsighted people since they wear diverging lenses.

24.40. **Model:** Assume thin lenses and treat each as a simple magnifier with $M = 25\text{cm}/f$.

Visualize: Equation 24.10 gives the magnification of a microscope.

$$M = m_{\text{obj}} M_{\text{eye}} = -\frac{L}{f_{\text{obj}}} \frac{25\text{cm}}{f_{\text{eye}}}$$

Solve: **(a)** The more powerful lens $(4\times)$ with the shorter focal length should be used as the objective.

(b) Solve the equation above for L (drop the negative sign).

$$L = \frac{M f_{obj} f_{eye}}{25 \text{ cm}} = \frac{(12)(\frac{25\text{ cm}}{4})(\frac{25\text{ cm}}{2})}{25 \text{ cm}} = 37.5 \text{ cm}$$

Assess: This is a long microscope tube.

24.41. Visualize: We'll use the thin-lens equation and also Equation 24.10:

$$M = -\frac{L}{f_{obj}}\frac{25\text{ cm}}{f_{eye}}$$

Also recall that $P = 1/f$.

Solve: The power of each lens is

$$P_1 = \frac{1}{f_1} = \frac{1}{0.020\text{ m}} = 50\text{ D} \qquad P_2 = \frac{1}{f_1} = \frac{2}{0.010\text{ m}} = 100\text{ D}$$

Since we want to use the more powerful lens as the objective, the lens labeled P_2 will be the objective. This means the focal lengths of the objective and eyepiece are $f_{obj} = 1.0$ cm and $f_{eye} = 2.0$ cm.

(a) We want the eyepiece to $L = 16$ cm from the objective, so $s' = 16\text{ cm} - f_{eye} = 16\text{ cm} - 2.0\text{ cm} = 14$ cm. The object distance for the objective is

$$s = \frac{fs'}{s' - f} = \frac{(1.0\text{ cm})(14\text{ cm})}{14\text{ cm} - 1.0\text{ cm}} = 1.1\text{ cm}$$

(b) $M = m_{obj}M_{eye} = \frac{-s'}{s}\frac{25\text{ cm}}{f_{eye}} = -\frac{(14\text{ cm})}{(1.08\text{ cm})}\frac{25\text{ cm}}{(2.0\text{ cm})} = -160$

Assess: As expected, s is just beyond the focal point. We can use approximation in Equation 24.9 to get a similar answer:

$$M = \frac{L}{f_{obj}}\frac{25\text{ cm}}{f_{eye}} = -\frac{(16\text{ cm})}{(1.0\text{ cm})}\frac{25\text{ cm}}{(2.0\text{ cm})} = -200$$

but the approximation isn't very good for this microscope.

24.42. Model: While $s \approx f_{obj}$ we will not assume they are equal.

Visualize: Equation 24.9 says $m_{obj} \approx -L/f_{obj}$. We are given $L = 180$ mm and $m_{obj} = -40$, where the negative sign means the image is inverted.

Solve: Solve for f_{obj}.

$$f_{obj} = -\frac{L}{m_{obj}} = \frac{180 \text{ mm}}{40} = 4.5 \text{ mm}$$

From Equation 24.8, $M_{eye} = (25 \text{ cm})/f_{eye} \Rightarrow f_{eye} = 25 \text{ cm}/20 = 1.25 \text{ cm}$. For relaxed eye viewing the image of the objective must be $1.25 \text{ cm} = 12.5 \text{ mm}$ from the eyepiece, so $s' = 180 \text{ mm} - 12.5 \text{ mm} = 167.5 \text{ mm}$. Thus the sample distance is

$$s = \left(\frac{1}{4.5 \text{ mm}} - \frac{1}{167.5 \text{ mm}}\right)^{-1} = 4.6 \text{ mm}$$

Assess: You need a short focal length to achieve $800\times$ magnification. We can also verify that $s \approx f_{obj}$.

24.43. Model: The width of the central maximum that accounts for a significant amount of diffracted light intensity is inversely proportional to the size of the aperture. The lens is an aperture that focuses light.

Solve: To focus a laser beam, which consists of parallel rays from $s = \infty$, the focal length needs to match the distance to the target: $f = L = 5.0$ cm. The minimum spot size to which a lens can focus is

$$w = \frac{2.44\lambda f}{D} \Rightarrow 5.0\times10^{-6}\text{ m} = \frac{2.44\left(1.06\times10^{-6}\text{ m}\right)\left(5.0\times10^{-2}\text{ m}\right)}{D} \Rightarrow D = 2.6\text{ cm.}$$

24.44. Model: Two objects are marginally resolved if the angular separation between the objects, as seen from your eye lens, is $\alpha = 1.22\lambda/D$, but the λ we want to use is the λ in the eye: $\lambda = \lambda_{\text{air}}/n$. Let Δx be the separation between the two headlights of the oncoming car and let L be the distance of these lights from your eyes. For small angles, $\Delta x = \alpha L$.

We are given $D = 7\times10^{-3}$ m, $\Delta x = 1.2$ m, and $\lambda = \lambda_{\text{air}}/n = 600\text{ nm}/1.33 = 450$ nm.

Solve: Let Δy be the separation between the two headlights of the incoming car and let L be the distance of these lights from your eyes. Then,

$$\alpha = \frac{\Delta x}{L} = \frac{1.20\text{ m}}{L} = \frac{1.22\lambda}{D} = \frac{1.22(450\text{ nm})}{(7.0\times10^{-3}\text{ m})} \Rightarrow L = \frac{(1.20\text{ m})(7.0\times10^{-3}\text{ m})}{(1.22)(450\times10^{-9}\text{ m})} = 15\text{ km}$$

Assess: The two headlights are not resolvable if $L > 15$ km, marginally resolvable at 15 km, and resolvable at $L < 15$ km.

24.45. **Visualize:** The angle subtended at the eye due to a circle of diameter d at a near point of 25 cm is $\alpha = d/25$ cm. The angle to the first dark minimum in a circular diffraction pattern is $\theta_1 = 1.22\lambda/D$, where $\lambda = \lambda_{\text{air}}/n$ is the wavelength of the light in the eye and D is the pupil diameter. To just barely see the circle as a circle the condition $\alpha = \theta_1$ must be met.

We are given $D = 2.0\times10^{-3}$ m and $\lambda = \lambda_{\text{air}}/n = 600 \text{ nm}/1.33 = 450$ mm.

Solve: Equating α and θ_1 we have $d/25 \text{ cm} = 1.22\lambda/D$, which may be solved for the diameter of the circle.

$$d = \frac{(25 \text{ cm})(1.22\lambda)}{D} = \frac{(25 \text{ cm})(1.22)(450\times10^{-9}\text{m})}{2.0\times10^{-3} \text{ m}} = 6.9\times10^{-5} \text{ m} = 0.069 \text{ mm}$$

Assess: The above number is about the size of a needle point.

24.46. Visualize: For telescopes the angular resolution is

$$\theta = \frac{1.22\lambda}{D}$$

And for small angles, $s = \theta r$. We want to know s.

We are given $\lambda = 650$ nm, $D = 2.4$ m, and $r = 33{,}000 \text{ ly} = 2.8 \times 10^{17}$ km.

Solve: Combine the two equations.

(a)

$$s = \theta r = \frac{1.22\lambda}{D} r = \frac{(1.22)(650 \times 10^{-9} \text{ m})}{2.4 \text{ m}} 2.8 \times 10^{17} \text{ km} = 9.4 \times 10^{10} \text{ km}$$

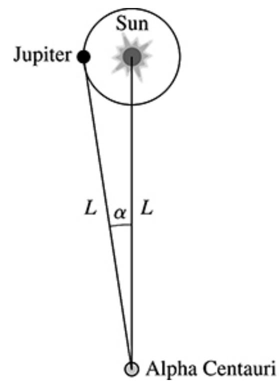
(b) The distance from the sun to Jupiter is 7.8×10^{11} m. So we divide the answer from part (**a**) by this number.

$$\frac{9.4 \times 10^{10} \text{ km}}{7.8 \times 10^{11} \text{ m}} = 120$$

Assess: The HST is good but not good enough to resolve two objects as close together as the sun and Jupiter from a distance of 30,000 ly.

24.47. Model: Two objects are marginally resolved if the angular separation between the objects is $\alpha = 1.22\lambda / D$.

Visualize:



Solve: **(a)** The angular separation between the sun and Jupiter is

$$\alpha = \frac{780\times 10^9 \text{ m}}{4.3 \text{ light years}} = \frac{780\times 10^9 \text{ m}}{4.3\times\left(3.0\times 10^8\right)\times\left(365\times 24\times 3600\right) \text{ m}} = 1.92\times 10^{-5} \text{ rad}$$

$$\alpha = \frac{1.22\lambda}{D} = \frac{1.22\left(600\times 10^{-9} \text{ m}\right)}{D} \Rightarrow D = 0.038 \text{ m} = 3.8 \text{ cm}$$

(b) The sun is vastly brighter than Jupiter, which is much smaller and seen only dimly by reflected light. In theory it may be possible to resolve Jupiter and the sun, but in practice the extremely bright light from the sun will overwhelm the very dim light from Jupiter.

24.48. Visualize: We'll start with Equation 24.15 and substitute in Equation 24.11 and an expression for α from Figure 24.14a.
We are given $f_{\text{obj}} = 1.6$ mm, $d_{\text{min}} = 400$ nm, $n = 1.0$, and $\lambda = 550$ nm.

Solve: Solve for *D*.

$$d_{\text{min}} = \frac{0.61\lambda}{\text{NA}} = \frac{0.61\lambda}{n\sin\alpha} = \frac{0.61\lambda}{n\sin\left(\tan^{-1}\dfrac{D/2}{f_{\text{obj}}}\right)} =$$

$$\sin\left(\tan^{-1}\frac{D/2}{f_{\text{obj}}}\right) = \frac{0.61\lambda}{n\,d_{\text{min}}}$$

$$\tan^{-1}\frac{D/2}{f_{\text{obj}}} = \sin^{-1}\frac{0.61\lambda}{n\,d_{\text{min}}}$$

$$\frac{D/2}{f_{\text{obj}}} = \tan\left(\sin^{-1}\frac{0.61\lambda}{n\,d_{\text{min}}}\right)$$

$$D = 2f_{\text{obj}}\tan\left(\sin^{-1}\frac{0.61\lambda}{n\,d_{\text{min}}}\right) = 2(1.6\text{ mm})\tan\left(\sin^{-1}\frac{0.61(550\text{ nm})}{(1.0)(400\text{ nm})}\right) = 4.9\text{ mm}$$

The diameter of the lens must exceed 4.9 mm.
Assess: A half centimeter is in the ballpark for microscope lens diameters.

24.49. Model: For a diffraction-limited lens, the minimum focal length is the same size as its diameter. The smallest spot diameter over which you can focus light is $w_{min} \approx 2.5\lambda$.

Solve: **(a)** The smallest spot size is $w_{min} \approx 2.5\lambda = 2.5(800 \times 10^{-9}\text{ m}) = 2\ \mu\text{m}$.

(b) The total usable area of the optical disk is

$$\pi\left[(5.5\times10^{-2}\text{ m})^2 - (2\times10^{-2}\text{ m})^2\right] = 0.00825\text{ m}^2$$

The area of each pit is the area of one bit of information and is $[(1.25)(2\ \mu\text{m})]^2 = [2.5\ \mu\text{m}]^2$. The area of 1 byte is 8 times this quantity and the area of 1 megabyte (MB) of information is 10^6 times more. This means the number of megabytes (MB) of data that can be stored on the disk is

$$\frac{0.00825\text{ m}^2}{8\times(2.5\times10^{-6}\text{ m})^2\times10^6\text{ MB}^{-1}} = 165\text{ MB}$$

Assess: A memory storage capacity of 165 MB is reasonable.

24.50. Visualize: Physically, the light rays can either go directly through the lens or they can reflect from the mirror and then go through the lens. We can consider the image from the lens alone and then consider the image from mirror becoming the object for the lens.

Solve:

(a) First case: the lens gets the subscript 1's and the mirror the 2's. The location of the image from the lens is

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(5\text{ cm})}{5\text{ cm} - 10\text{ cm}} = -10\text{ cm}$$

The image is right at the mirror plane and a calculation for a mirror shows that when $s_2 = 0$ then $s_2' = 0$, too. So the final image is at the mirror, 10 cm to the left of the lens.

$$m = -\frac{s_1'}{s_1} = -\frac{-10\text{ cm}}{5\text{ cm}} = 2.0$$

so $h' = hm = (1.0\text{ cm})(2.0) = 2.0\text{ cm}$.

Second case: the mirror gets the subscript 1's and the lens the 2's. The location of the image from the mirror is

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(10\text{ cm})(5\text{ cm})}{5\text{ cm} - 10\text{ cm}} = -10\text{ cm}$$

or 10 cm behind (to the left of) the mirror. This image now becomes the object for the lens and $s_2 = 20$ cm.

$$s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(10\text{ cm})(20\text{ cm})}{20\text{ cm} - 10\text{ cm}} = 20\text{ cm}$$

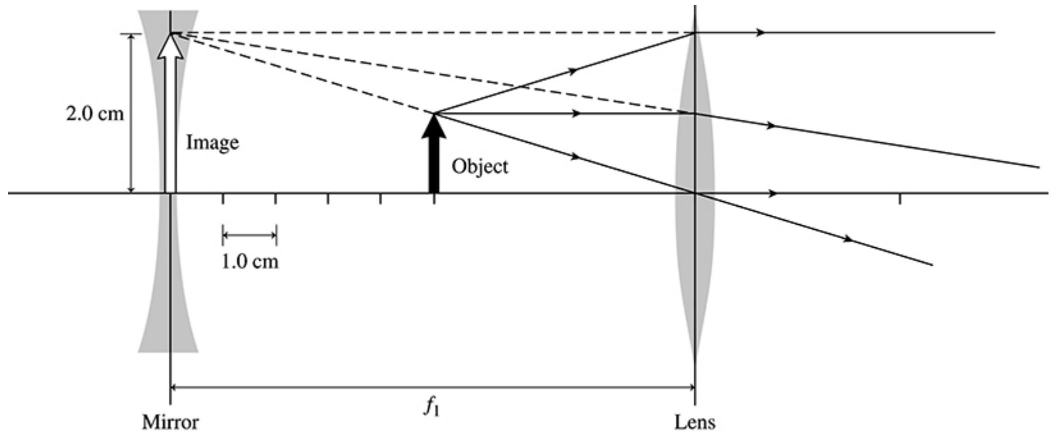
So the image is 20 cm to the right of the lens.

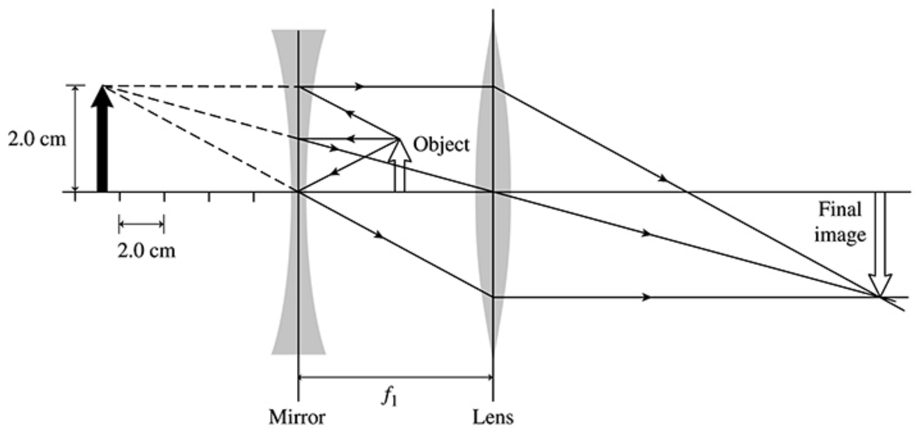
$$m = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = \left(-\frac{-10\text{ cm}}{5\text{ cm}}\right)\left(-\frac{20\text{ cm}}{20\text{ cm}}\right) = -2.0$$

so $h' = hm = (1.0\text{ cm})(-2.0) = -2.0\text{ cm}$, where the negative sign indicates the image is inverted.

In summary, both images are 2.0 cm tall; one is upright 10 cm left of the lens, the other is inverted 20 cm to the right of the lens.

(b)





Assess: The ray tracing verifies the calculations.

24.51. Model: In the small angle approximation the angle subtended by Mars without the telescope is $\theta_{\text{obj}} = D/d$ where D is the diameter and d is the distance from the earth.

Visualize: We are given $f_{\text{eye}} = 2.5$ cm.

Solve:

$$M = \frac{\theta_{\text{eye}}}{\theta_{\text{obj}}} = \frac{0.50°}{D/d} = \frac{0.50°}{6800 \times 10^3 \text{ m}/1.1 \times 10^{11} \text{ m}} \left(\frac{\pi \text{ rad}}{180°}\right) = 141$$

Ignoring the negative sign,

$$M = \frac{f_{\text{obj}}}{f_{\text{eye}}}$$

$$f_{\text{obj}} = Mf_{\text{eye}} = 141(2.5 \text{ cm}) = 353 \text{ cm}$$

The length of the telescope is

$$L = f_{\text{obj}} + f_{\text{eye}} = 352.5 \text{ cm} + 2.5 \text{ cm} = 355 \text{ cm} = 3.55 \text{ m} \approx 3.5 \text{ m}$$

Assess: This is longer than most amateur telescopes.

24.52. Visualize: The plane left face will not refract any of the rays (which are parallel to each other and perpendicular to the face), so nothing happens until the rays hit the first curved surface between lens 1 and lens 2. We'll need to twice (once for each curved surface) apply Equation 23.21:

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$$

Solve: **(a)** For the first curved surface we say $s=\infty$ because the incoming rays are parallel.

$$\frac{n_1}{\infty}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}\Rightarrow s'=R\frac{n_2}{n_2-n_1}$$

For the second curved surface (where the rays exit into the air) n_2 is on the left and $n_{\text{air}}=1.0$ is on the right (so those will take the place of n_1 and n_2, respectively). Since the distance between the lenses is zero, s' from the previous result will be plugged in for s for the second case. The final thing to note is that the magnitude of the new $s'=f$ because the doublet brings parallel rays to a focus at f, but s' is negative due to the sign convention in Table 23.3.

$$\frac{n_2}{R\frac{n_2}{n_2-n_1}}+\frac{n_{\text{air}}}{-f}=\frac{n_{\text{air}}-n_2}{R}$$

Now solve for f.

$$\frac{n_2-n_1}{R}+\frac{1}{-f}=\frac{1-n_2}{R}$$

$$\frac{1}{-f}=\frac{1-n_2-n_2+n_1}{R}$$

$$f=\frac{R}{2n_2-n_1-1}$$

(b)

$$f_{\text{blue}}=\frac{R}{2(n_2)_{\text{blue}}-(n_1)_{\text{blue}}-1}\qquad f_{\text{red}}=\frac{R}{2(n_2)_{\text{red}}-(n_1)_{\text{red}}-1}$$

In the condition we desire $f_{\text{blue}}=f_{\text{red}}$, so the two denominators must be equal.

$$2(n_2)_{\text{blue}}-(n_1)_{\text{blue}}-1=2(n_2)_{\text{red}}-(n_1)_{\text{red}}-1$$

$$2\left[(n_2)_{\text{blue}}-(n_1)_{\text{red}}\right]=(n_1)_{\text{blue}}-(n_1)_{\text{red}}$$

$$\Delta n_2=\frac{1}{2}\Delta n_1$$

(c) Simply find the Δn for each type of glass and hope one is twice the other.

$$\Delta n_{\text{crown}}=1.525-1.517=0.008$$

$$\Delta n_{\text{flint}}=1.632-1.616=0.016$$

Since $\Delta n_{crown}=\frac{1}{2}\Delta n_{\text{flint}}$ then crown glass must be the second material, or the converging lens, while flint glass must be the first material, or the diverging lens.

(d) Solve the original focal length expression for R.

$$R=f\left(2n_2-n_1-1\right)$$

Since $f_{\text{blue}} = f_{\text{red}}$, it doesn't matter which color we choose for the n's (as long as we are consistent). Say we pick blue, so $n_1 = 1.632$ and $n_2 = 1.525$. We are given $f = 10.0$ cm.

$$R = (10.0 \text{ cm})\left[2(1.525) - 1.632 - 1\right] = 4.18 \text{ cm}$$

Assess: The answers to the various parts fit together and the final result is reasonable.

24.53. Visualize: The effective focal length is defined as the distance from the midpoint between the two lenses to the point that initially parallel rays come to a focus. Because $d < f$ the image from the first lens is to the right of the second lens, so we will use a negative object distance when we analyze the second lens.

Solve: **(a)** If the original object distance is very large $(s_1 \approx \infty)$ then $s_1' = f$. This image is to the right of the right lens by an amount $f - d$, but since we are treating this as a negative object distance we will put in $s_2 = -(f - d) = d - f$. The thin-lens equation for the second lens (the diverging one, with a negative focal length) becomes:

$$\frac{1}{d-f} + \frac{1}{s_2'} = \frac{1}{-f}$$

Solve for s_2'.

$$s_2' = \frac{-f(d-f)}{(d-f)-(-f)} = \frac{f_2 - fd}{d}$$

This is the distance of the focus to the right of the second lens, however, we want the distance from the midpoint between the lenses, so we add $\frac{1}{2}d$ to the answer.

$$f_{\text{eff}} = s_2' + \frac{1}{2}d = \frac{f^2 - fd}{d} + \frac{1}{2}d = \frac{f_2 - fd}{d} + \frac{\frac{1}{2}d^2}{d} = \frac{f^2 - fd + \frac{1}{2}d^2}{d}$$

(b) We'll plug $d = \frac{1}{2}f$ and $d = \frac{1}{4}f$ in turn into the previous result. Then Equation 24.1 shows that if we take the ratios of the resulting f_{eff} 's we'll have the zoom.

$$\text{zoom} = \frac{(f_{\text{eff}})_{d=1/4f}}{(f_{\text{eff}})_{d=1/2f}} = \frac{\left(\frac{f^2 - f(\frac{1}{4}f) + \frac{1}{2}(\frac{1}{4}f)^2}{\frac{1}{4}f}\right)}{\left(\frac{f^2 - f(\frac{1}{2}f) + \frac{1}{2}(\frac{1}{2}f)^2}{\frac{1}{2}f}\right)} = \left(\frac{\frac{1}{2}}{\frac{1}{4}}\right)\frac{f^2 - \frac{1}{4}f^2 + \frac{1}{32}f^2}{f^2 - \frac{1}{2}f^2 + \frac{1}{8}f^2}$$

Cancel f^2 from each term.

$$\left(\frac{4}{2}\right)\frac{1 - \frac{1}{4} + \frac{1}{32}}{1 - \frac{1}{2} + \frac{1}{8}} = (2)\frac{\frac{25}{32}}{\frac{5}{8}} - (2)\frac{5}{4} = \frac{5}{2} = 2.5$$

So the lens is a $2.5\times$ zoom lens.

Assess: This is a reasonable amount of zoom. The magnification spans a factor of 2.5.