23.1. **Model:** Light rays travel in straight lines.
Solve: **(a)** The time is

$$t = \frac{\Delta x}{c} = \frac{1.0 \text{ m}}{3 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-9} \text{ s} = 3.3 \text{ ns}$$

(b) The refractive indices for water, glass, and cubic zirconia are 1.33, 1.50, and 1.96, respectively. In a time of 3.33 ns, light will travel the following distance in water:

$$\Delta x_{water} = v_{water}t = \left(\frac{c}{n_{water}} \right) t = \left(\frac{3 \times 10^8 \text{ m/s}}{1.33} \right) \left(3.33 \times 10^{-9} \text{ s} \right) = 0.75 \text{ m}$$

Likewise, the distances traveled in the glass and cubic zirconia are $\Delta x_{glass} = 0.667$ m and $\Delta x_{cubic \ zirconia} = 0.458$ m.

Assess: The higher the refractive index of a medium, the slower the speed of light and hence smaller the distance it travels in that medium in a given time.

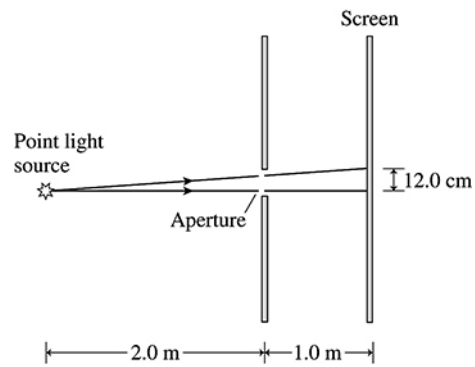**23.2.** **Model:** Light rays travel in straight lines.

Solve: Let t_{glass}, t_{oil}, and t_{plastic} be the times light takes to pass through the layers of glass, oil, and plastic. The time for glass is

$$t_{\text{glass}} = \frac{\Delta x}{v_{\text{glass}}} = \frac{\Delta x}{c/n_{\text{glass}}} = \frac{\Delta x\, n_{\text{glass}}}{c} = \frac{(1.0 \times 10^{-2}\text{ m})(1.50)}{3.0 \times 10^8\text{ m/s}} = 0.050\text{ ns}$$

Likewise, $t_{\text{oil}} = 0.243$ ns and $t_{\text{plastic}} = 0.106$ ns. Thus, $t_{\text{total}} = t_{\text{glass}} + t_{\text{oil}} + t_{\text{plastic}} = 0.050$ ns + 0.243 ns + 0.106 ns = 0.399 ns = 0.40 ns.

Assess: The small time is due to the high value for the speed of light.
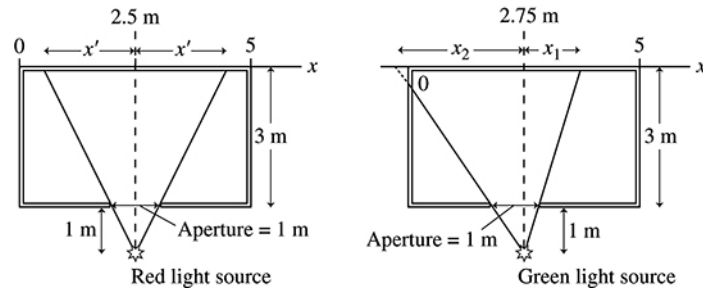
23.3. **Model:** Light rays travel in straight lines. The light source is a point source.
Visualize:

Solve: Let w be the width of the aperture. Then from the geometry of the figure,

$$\frac{w}{2.0\ \text{m}} = \frac{12.0\ \text{cm}}{2.0\ \text{m} + 1.0\ \text{m}} \Rightarrow w = 8.0\ \text{cm}$$

23.4. **Model:** Light rays travel in straight lines. Also, the red and green light bulbs are point sources.
Visualize:

Solve: The width of the aperture is $w = 1$ m. From the geometry of the figure for red light,
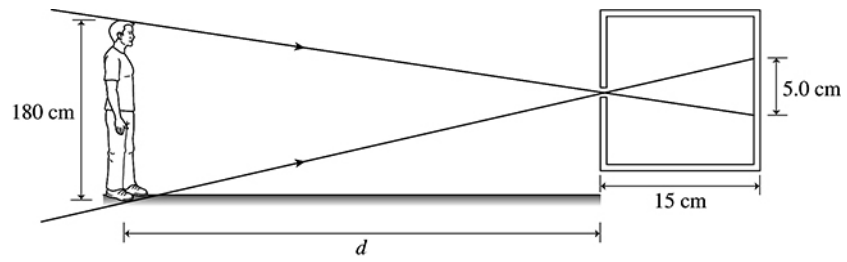
$$\frac{w/2}{1\text{ m}} = \frac{x'}{3\text{ m} + 1\text{ m}} \Rightarrow x' = 2w = 2(1.0\text{ m}) = 2.0\text{ m}$$

The red light illuminates the wall from $x = 0.50$ m to $x = 4.50$ m. For the green light,

$$\frac{w/4}{1\text{ m}} = \frac{x_1}{3\text{ m} + 1\text{ m}} \Rightarrow x_1 = 1.0\text{ m} \qquad \frac{3w/4}{1\text{ m}} = \frac{x_2}{3\text{ m} + 1\text{ m}} \Rightarrow x_2 = 3.0\text{ m}$$

Because the back wall exists only for 2.75 m to the left of the green light source, the green light has a range from $x = 0$ m to $x = 3.75$ m.

23.5. **Visualize:** Note the similar triangles in this figure.
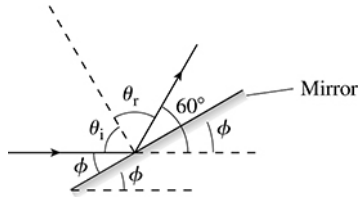

Solve:

$$\frac{15 \text{ cm}}{5 \text{ cm}} = \frac{d}{180 \text{ cm}}$$

$$d = \frac{15 \text{ cm}}{5 \text{ cm}}(180 \text{ cm}) = 540 \text{ cm} = 5.4 \text{ m}$$

Assess: This is a typical distance for photographs of people.

23.6. Model: Use the ray model of light.
Visualize:

According to the law of reflection, $\theta_r = \theta_i$.
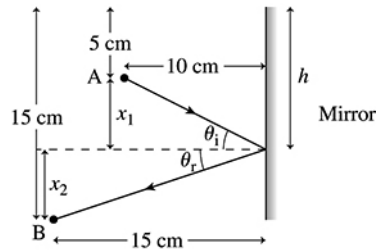**Solve:** From the geometry of the diagram,

$$\theta_i + \phi = 90° \qquad \theta_r + (60° - \phi) = 90°$$

Using the law of reflection, we get

$$90° - \phi = 90° - (60° - \phi) \implies \phi = 30°$$

Assess: The above result leads to a general result for plane mirrors: If a plane mirror rotates by an angle ϕ relative to the horizontal, the reflected ray makes an angle of 2ϕ with the horizontal.

23.7. **Model:** Light rays travel in straight lines and follow the law of reflection.
Visualize:

Solve: We are asked to obtain the distance $h = x_1 + 5.0$ cm. From the geometry of the diagram,

$$\tan\theta_i = \frac{x_1}{10 \text{ cm}} \qquad \tan\theta_r = \frac{x_2}{15 \text{ cm}} \qquad x_1 + x_2 = 10 \text{ cm}$$

Because $\theta_r = \theta_i$, we have

$$\frac{x_1}{10 \text{ cm}} = \frac{x_2}{15 \text{ cm}} = \frac{10 \text{ cm} - x_1}{15 \text{ cm}} \Rightarrow (15 \text{ cm})x_1 = 100 \text{ cm}^2 - 10x_1 \Rightarrow x_1 = 4.0 \text{ cm}$$

Thus, the ray strikes a distance 9.0 cm below the top edge of the mirror.

23.8. **Model:** Think of the view in the figure as a horizontal view of a vertical wall and the laser beam and hexagonal mirror in a vertical plane for ease of labeling. The laser beam will strike the highest spot on the wall when a new corner rotates into the laser beam and the angle the laser makes with the normal is greatest. We will compute how high on the wall this highest spot is from the center spot behind the laser; then we will multiply by two because symmetry says the reflected beam will hit the lowest spot just as the face rotates out of the laser beam (and the beam makes the largest angle with the normal in the downward direction), and then a new corner enters the beam with the reflection at the top again.

Visualize: From the small right triangle inside the hexagon we deduce $d = 0.20$ m$/\sin 60°$. Therefore, the distance from the wall to the corner of the hexagon just as it enters the laser beam is 2.0 m $- d$. This becomes the base of a large right triangle whose side on the wall is x and whose angle opposite x is $60°$.
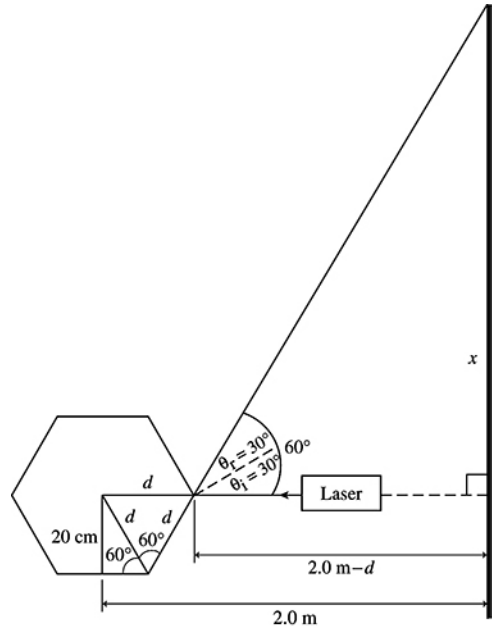
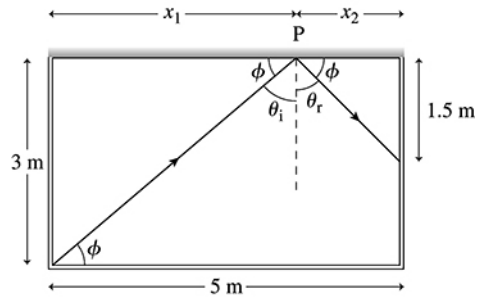**Solve:** Solve the large right triangle for x.

$$\tan 60° = \frac{x}{2.0 \text{ m} - d}$$

$$x = (\tan 60°)(2.0 \text{ m} - d) = (\tan 60°)\left(2.0 \text{ m} - \frac{0.20 \text{ m}}{\sin 60°} \right) = 3.06 \text{ m}$$

Now, because of symmetry, double x to get the total length of the streak of laser light: $2x = 6.1$ m.

Assess: The 50 cm distance from the laser to the center of the hexagon is irrelevant.

23.9. **Model:** Light rays travel in straight lines and follow the law of reflection.
Visualize:

To determine the angle ϕ, we must know the point P on the mirror where the ray is incident. P is a distance x_2 from the far wall and a horizontal distance x_1 from the laser source. The ray from the source must strike P so that the angle of incidence θ_i is equal to the angle of reflection θ_r.
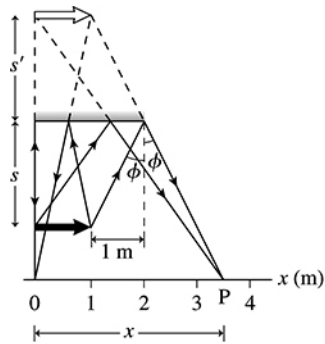**Solve:** From the geometry of the diagram,

$$\tan\phi = \frac{1.5 \text{ m}}{x_2} = \frac{3 \text{ m}}{x_1} \qquad x_1 + x_2 = 5 \text{ m}$$

$$\Rightarrow \frac{1.5 \text{ m}}{5 \text{ m} - x_1} = \frac{3 \text{ m}}{x_1} \Rightarrow (1.5 \text{ m})x_1 = 15 \text{ m}^2 - (3 \text{ m})x_1 \Rightarrow x_1 = \frac{10}{3} \text{ m}$$

$$\Rightarrow \tan\phi = \frac{3 \text{ m}}{x_1} = \frac{9}{10} = 0.90 \Rightarrow \phi = 42°$$

23.10. **Model:** Use the ray model of light.

Visualize:

The arrow is at a distance s from the mirror, so its image is at a distance s' behind the mirror. When you are at $x = 0$ m, a ray from the arrow's head, after reflection from the mirror, is able to enter your eye. Similarly, a ray from the arrow's tail, after normal incidence, is reflected into the eye. That is, the eye is able to see the arrow's head and tail. While walking toward the right, a ray from the arrow's head will reflect from the mirror's right edge and enter your eye at P. A ray starting from the arrow's tail will also enter your eye when you are at P. That is, while at P you will be able to see the entire image of the arrow. However, the light from the arrow's head can never reach beyond point P.
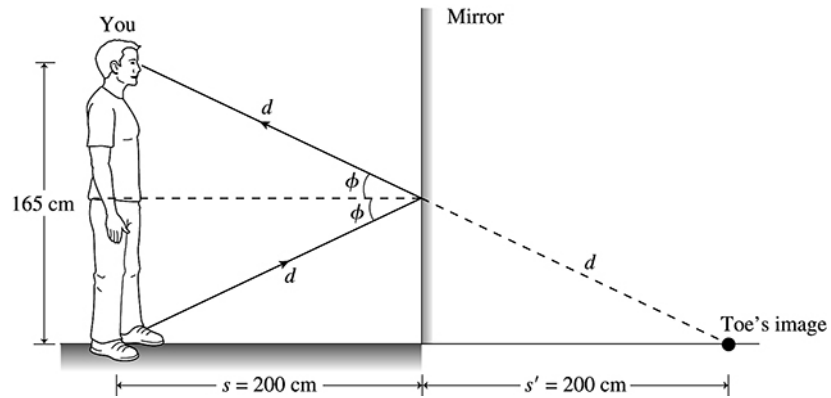
Solve: Point P is a distance x from the origin. From the geometry of the diagram,

$$\tan\phi = \frac{1\text{ m}}{s} = \frac{1\text{ m}}{2\text{ m}} = \frac{x-2\text{ m}}{s+1\text{ m}} = \frac{x-2\text{ m}}{3\text{ m}} \Rightarrow \frac{1}{2} = \frac{x-2\text{ m}}{3\text{ m}} \Rightarrow x = 3.5\text{ m}$$

Thus, the range of x over which you can see the entire arrow in the mirror is 0 m to 3.5 m.

23.11. **Model:** Use the ray model of light and the law of reflection.
Visualize:

We only need one ray of light that leaves your toes and reflects in your eye.
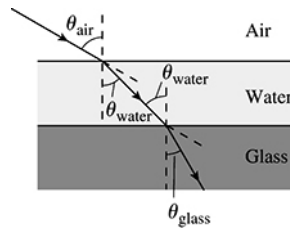**Solve:** From the geometry of the diagram, the distance from your eye to the toes' image is

$$2d = \sqrt{(400 \text{ cm})^2 + (165 \text{ cm})^2} = 433 \text{ cm}$$

Assess: The light appears to come from your toes' image.

23.12. **Model:** Use the ray model of light and Snell's law.

Visualize:

Solve: According to Snell's law for the air-water and water-glass boundaries,

$$n_{air} \sin \theta_{air} = n_{water} \sin \theta_{water} \qquad n_{water} \sin \theta_{water} = n_{glass} \sin \theta_{glass}$$

From these two equations, we have

$$n_{air} \sin \theta_{air} = n_{glass} \sin \theta_{glass} \Rightarrow \sin \theta_{glass} = \frac{n_{air}}{n_{glass}} \sin \theta_{air} = \left(\frac{1.0}{1.50} \right) \sin 60° \Rightarrow \theta_{glass} = \sin^{-1} \left(\frac{\sin \; 60°}{1.5} \right) = 35°$$

23.13. **Visualize:** Use Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$. We are given $\theta_2 = \theta_{CZ} = 25°$. We look up in Table 23.1 $n_1 = n_{oil} = 1.46$ and $n_2 = n_{CZ} = 1.96$.

Solve: Solve the equation for θ_1.

$$\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.96}{1.46}\sin 25°\right) = 35°$$

Assess: Since the ray goes into a material with higher index of refraction we know it bends toward the normal, so we expect $\theta_1 > \theta_2$; this is the case.

23.14. **Model:** Use the ray model of light. The sun is a point source of light.
Visualize:
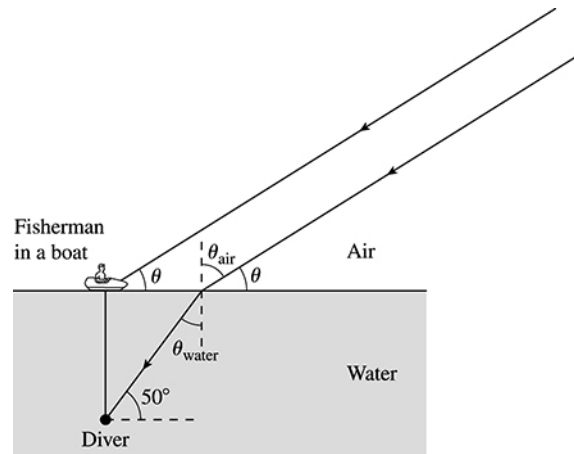
A ray that arrives at the diver 50° above horizontal refracted into the water at $\theta_{water} = 40°$.
Solve: Using Snell's law at the water-air boundary

$$n_{air} \sin \theta_{air} = n_{water} \sin \theta_{water} \Rightarrow \sin \theta_{air} = \frac{n_{water}}{n_{air}} \sin \theta_{water} = \left(\frac{1.33}{1.0}\right) \sin 40°$$

$$\Rightarrow \theta_{air} = 58.7°$$

Thus the height above the horizon is $\theta = 90° - \theta_{air} = 31.3° \approx 31°$. Because the sun is far away from the fisherman (and the diver), the fisherman will see the sun at the same angle of $31°$ above the horizon.

**23.15.** **Model:** Represent the laser beam with a single ray and use the ray model of light.
Solve: Using Snell's law at the air-water boundary,

$$n_{air} \sin\theta_{air} = n_{liquid} \sin\theta_{liquid} \Rightarrow n_{liquid} = n_{air} \frac{\sin\theta_{air}}{\sin\theta_{liquid}} = 1.0\left(\frac{\sin 37°}{\sin 26°}\right) = 1.37$$

Assess: As expected, n_{liquid} is larger than n_{air}.

23.16. **Model:** Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.

Visualize:
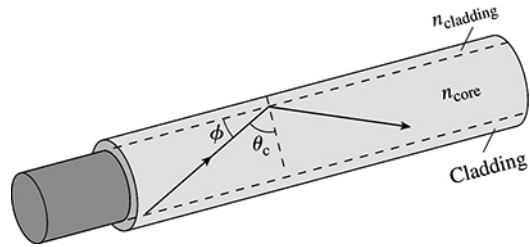
Solve: The critical angle of incidence is given by Equation 23.9:

$$\theta_c = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\right) = \sin^{-1}\left(\frac{1.48}{1.60}\right) = 67.7°$$

Thus, the maximum angle a light ray can make with the wall of the core to remain inside the fiber is $90° - 67.7°$ = 23.3°.

Assess: We can have total internal reflection because $n_{core} > n_{cladding}$.

23.17. **Model:** Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.

Visualize:

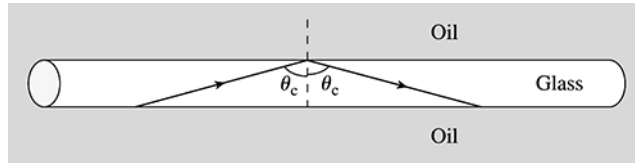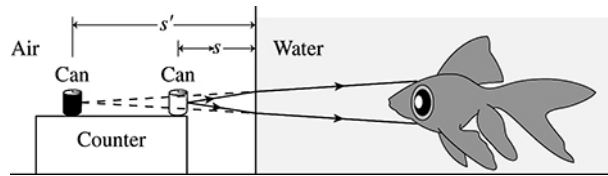**Solve:** The critical angle of incidence is given by Equation 23.9:

$$\theta_c = \sin^{-1}\left(\frac{n_{oil}}{n_{glass}}\right) = \sin^{-1}\left(\frac{1.46}{1.50}\right) = 76.7°$$

Assess: The critical angle exists because $n_{oil} < n_{glass}$.

23.18. **Model:** Represent the can as a point source and use the ray model of light.
Visualize:

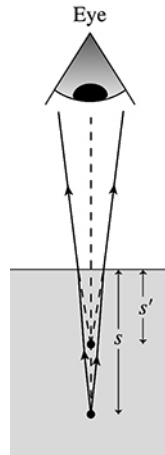Paraxial rays from the can refract into the water and enter into the fish's eye.
Solve: The object distance from the edge of the aquarium is s. From the water side, the can appears to be at an image distance $s' = 30$ cm. Using Equation 23.13,

$$s' = \frac{n_2}{n_1}s = \frac{n_{water}}{n_{air}}s = \left(\frac{1.33}{1.0}\right)s \ \Rightarrow \ s = \frac{30 \text{ cm}}{1.33} = 23 \text{ cm}$$

23.19. **Model:** Represent the beetle as a point source and use the ray model of light.
Visualize:

Eye

Paraxial rays from the beetle refract into the air and then enter into the observer's eye. The rays in the air when extended into the plastic appear to be coming from the beetle at a shallower location, a distance s' from the plastic-air boundary.

Solve: The actual object distance is s and the image distance is $s' = 2.0$ cm. Using Equation 23.13,

$$s' = \frac{n_2}{n_1}s = \frac{n_{air}}{n_{plastic}}s \Rightarrow 2.0 \text{ cm} = \frac{1.0}{1.59}s \Rightarrow s = 3.2 \text{ cm}$$

Assess: The beetle is much deeper in the plastic than it appears to be.

23.20. Model: Represent the diver's head and toes as point sources. Use the ray model of light.
Visualize:
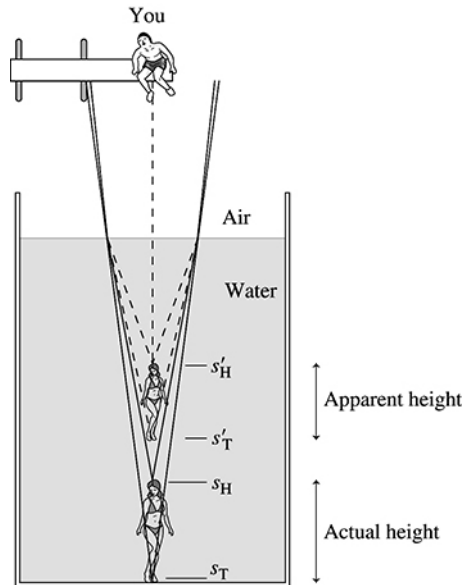
Paraxial rays from the head and the toes of the diver refract into the air and then enter into your eyes. When these refracted rays are extended into the water, the head and the toes appear elevated toward you.
Solve: Using Equation 23.13,

$$s'_T = \frac{n_2}{n_1} s_T = \frac{n_{air}}{n_{water}} s_T \qquad\qquad s'_H = \frac{n_{air}}{n_{water}} s_H$$

Subtracting the two equations, her apparent height is

$$s'_H - s'_T = \frac{n_{air}}{n_{water}}(s_H - s_T) = \frac{1.0}{1.33}(150 \text{ cm}) = 113 \text{ cm}$$

23.21. Model: Represent the aquarium's wall as a point source, and use the ray model of light.
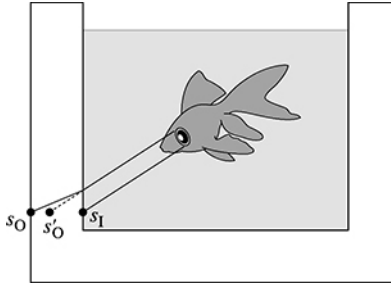**Visualize:**

Paraxial rays from the outer edge (O) are refracted into the water and then enter into the fish's eye. When extended into the wall, these rays will appear to be coming from O′ rather from O. The point on the inside edge (I) of the wall will not change its apparent location.

Solve: We are given that $s_O - s_I = 4.00$ mm and $s'_O - s'_I = 3.50$ mm. Using Equation 23.13,

$$s'_O = \frac{n_{\text{water}}}{n_{\text{wall}}} s_O \qquad s'_I = \frac{n_{\text{water}}}{n_{\text{wall}}} s_I$$

$$\Rightarrow s'_O - s'_I = \frac{n_{\text{water}}}{n_{\text{wall}}}(s_O - s_I) \Rightarrow 3.50 \text{ mm} = \frac{1.33}{n_{\text{wall}}}(4.00 \text{ mm}) \Rightarrow n_{\text{wall}} = (1.33)\left(\frac{4.00 \text{ mm}}{3.50 \text{ mm}}\right) = 1.52$$

23.22. **Model:** Use the ray model of light.
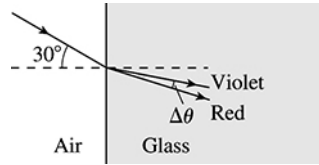
Visualize:

Solve: Using Snell's law,

$$n_{air} \sin 30° = n_{red} \sin \theta_{red} \implies \theta_{red} = \sin^{-1}\left(\frac{\sin 30°}{1.52}\right) = 19.2°$$

$$n_{air} \sin 30° = n_{violet} \sin \theta_{violet} \implies \theta_{violet} = \sin^{-1}\left(\frac{\sin 30°}{1.55}\right) = 18.8°$$

Thus the angular spread is

$$\Delta\theta = \theta_{red} - \theta_{violet} = 19.2° - 18.8° = 0.4°$$

23.23. Model: Use the ray model of light and the phenomenon of dispersion.
Visualize:

Solve: **(a)** From the graph in Figure 23.29, we estimate the index of refraction for the red light (656 nm) to be n_{red} = 1.572 and for the blue light (456 nm) to be n_{blue} = 1.587.
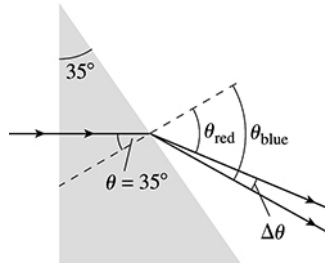**(b)** The angle of incidence onto the rear of the prism is 35°. Using these values for the refractive index and Snell's law,

$$n_{\text{red}} \sin 35° = n_{\text{air}} \sin \theta_{\text{red}} \Rightarrow \theta_{\text{red}} = \sin^{-1}\left(\frac{1.572 \sin 35°}{1.0}\right) = 64.4°$$

$$n_{\text{blue}} \sin 35° = n_{\text{air}} \sin \theta_{\text{blue}} \Rightarrow \theta_{\text{blue}} = \sin^{-1}\left(\frac{1.587 \sin 35°}{1.0}\right) = 65.5°$$

$$\Rightarrow \Delta\theta = \theta_{\text{blue}} - \theta_{\text{red}} = 1.1°$$

23.24. **Model:** Use the ray model of light and the phenomenon of dispersion.
Visualize:
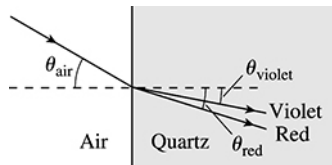
Solve: Using Snell's law for the red light,

$$n_{air} \sin \theta_{air} = n_{red} \sin \theta_{red} \Rightarrow 1.0 \sin \theta_{air} = 1.45 \sin 26.3° \Rightarrow \theta_{air} = \sin^{-1}(1.45 \sin 26.3°) = 40.0°$$

Now using Snell's law for the violet light,

$$n_{air} \sin \theta_{air} = n_{violet} \sin \theta_{violet} \Rightarrow 1.0 \sin 40.0° = n_{violet} \sin 25.7° \Rightarrow n_{violet} = 1.48$$

Assess: As expected, n_{violet} is slightly larger than n_{red}.

23.25. **Model:** The intensity of scattered light is inversely proportional to the fourth power of the wavelength.

Solve: We want to find the wavelength of infrared light such that $I_{IR} = 0.01 I_{500}$. Because $I_{500} \propto (500 \text{ nm})^{-4}$ and $I_{IR} \propto \lambda^{-4}$, we have

$$\frac{I_{500}}{I_{IR}} = \left(\frac{\lambda}{500 \text{ nm}}\right)^4 = 100 \Rightarrow \lambda = 1580 \text{ nm}$$

23.26. **Model:** Use ray tracing to locate the image.
Solve:
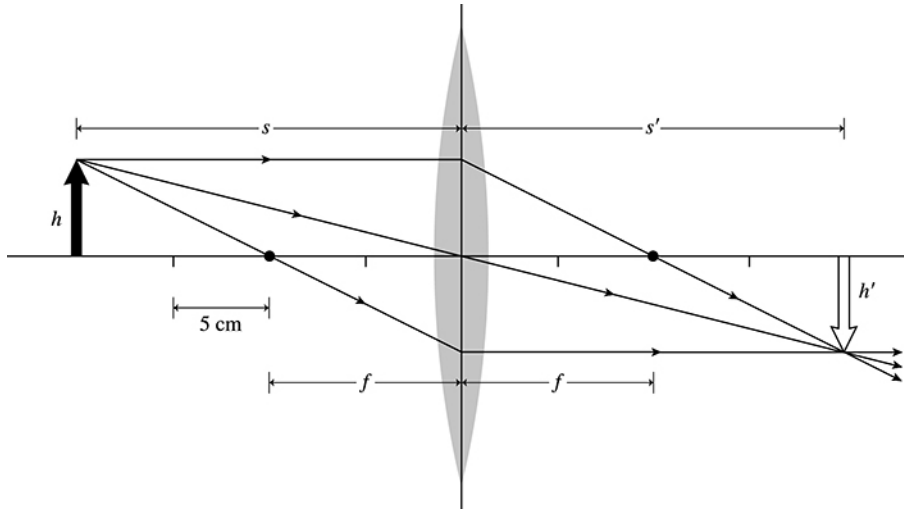
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. You can see from the diagram that the image is in the plane where the three special rays converge. The image is inverted and is located at $s' = 20.0$ cm to the right of the converging lens.

23.27. **Model:** Use ray tracing to locate the image.
Solve:
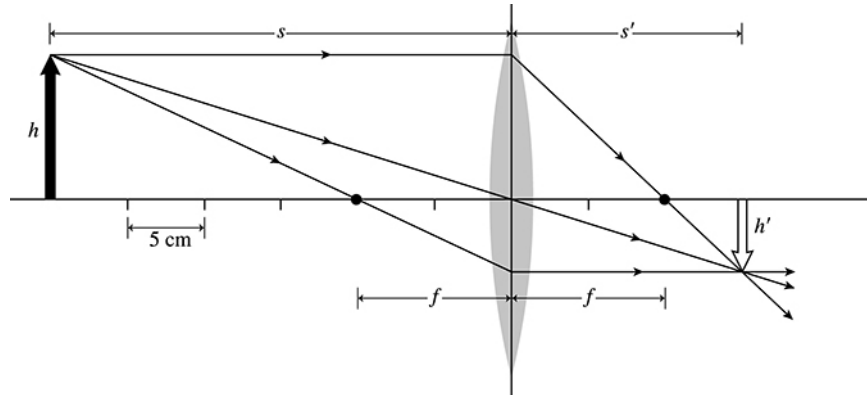
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. You can see from the diagram that the image is in the plane where the three special rays converge. The image is located at $s' = 15$ cm to the right of the converging lens, and is inverted.

23.28. **Model:** Use ray tracing to locate the image.

Solve:
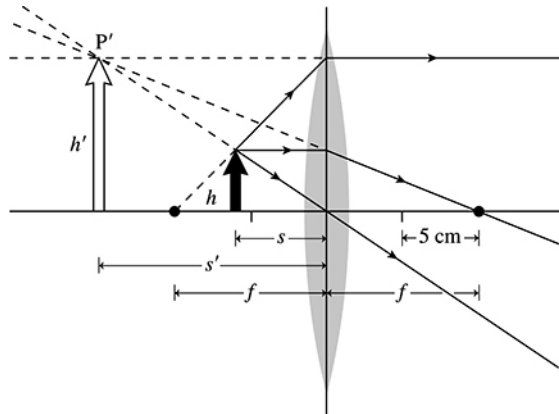
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. You can see that the rays after refraction do not converge at a point on the refraction side of the lens. On the other hand, the three special rays, when extrapolated backward toward the incidence side of the lens, meet at P′, which is 15 cm from the lens. That is, $s' = -15$ cm. The image is upright.

23.29. **Model:** Use ray tracing to locate the image.
Solve:
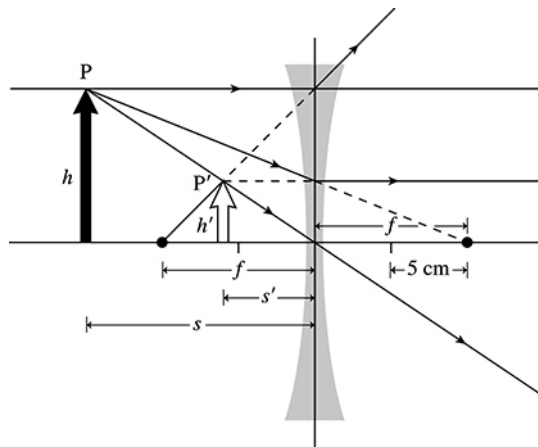
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.3. The three rays after refraction do not converge at a point, but they appear to come from P′. P′ is 6 cm from the diverging lens, so $s' = -6$ cm. The image is upright.

23.30. **Model:** Assume the biconvex lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = +40$ cm (convex toward the object) and the second surface has $R_2 = -40$ cm (concave toward the object). The index of refraction of glass is $n = 1.50$, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{40 \text{ cm}} - \frac{1}{-40 \text{ cm}}\right) \Rightarrow f = 40 \text{ cm}$$

23.31. **Model:** Assume the planoconvex lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = \infty$ and the second surface has $R_2 = -40$ cm (concave toward the object). The index of refraction of polystyrene plastic is 1.59, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.59-1)\left(\frac{1}{\infty} - \frac{1}{-40 \text{ cm}}\right) \Rightarrow \frac{1}{f} = \frac{0.59}{40 \text{ cm}} \Rightarrow f = 68 \text{ cm}$$

23.32. **Model:** Assume the biconcave lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = -40$ cm (concave toward the object) and the second surface has $R_2 = +40$ cm (convex toward the object). The index of refraction of glass is 1.50, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{-40 \text{ cm}} - \frac{1}{+40 \text{ cm}}\right) = (0.50)\left(-\frac{1}{20 \text{ cm}}\right) \Rightarrow f = -40 \text{ cm}$$

23.33. **Model:** Assume the meniscus lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = 30$ cm (convex toward the object) and the second surface has $R_2 = 40$ cm (convex toward the object). The index of refraction of polystyrene plastic is 1.59, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.59-1)\left(\frac{1}{30 \text{ cm}} - \frac{1}{40 \text{ cm}}\right) \Rightarrow f = 203 \text{ cm} \approx 200 \text{ cm}$$

23.34. **Model:** The water is a spherical refracting surface. Consider the paraxial rays that refract from the air into the water.

Solve: If the cat's face is 20 cm from the edge of the bowl, then $s = +20$ cm. The spherical fish bowl surface has $R = +25$ cm, because it is the convex surface that is toward the object. Also $n_1 = 1$ (air) and $n_2 = 1.33$ (water). Using Equation 23.21,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{20 \text{ cm}} + \frac{1.33}{s'} = \frac{1.33 - 1}{25 \text{ cm}} = \frac{0.33}{25 \text{ cm}} = 0.0132 \text{ cm}^{-1}$$

$$\Rightarrow \frac{1.33}{s'} = (0.0132 - 0.050) \text{ cm}^{-1} \Rightarrow s' = -36 \text{ cm}$$

This is a virtual image located 36 cm outside the fishbowl. The fish, inside the bowl, sees the virtual image. That is, the fish sees the cat's face 36 cm from the bowl.

23.35. **Model:** Model the bubble as a point source and consider the paraxial rays that refract from the plastic into the air. The edge of the plastic is a spherical refracting surface.

Visualize:
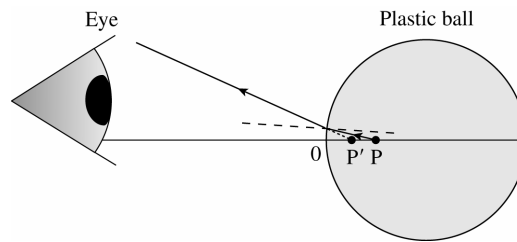
Solve: The bubble is at P, a distance of 2.0 cm from the surface. So, $s = 2.0$ cm. A ray from P after refracting from the plastic-air boundary bends away from the normal axis and enters the eye. This ray appears to come from P′, so the image of P is at P′ and it is a virtual image. Because P faces the concave side of the refracting surface, $R = -4.0$ cm. Furthermore, $n_1 = 1.59$ and $n_2 = 1.0$. Using Equation 23.21,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.59}{2.0 \text{ cm}} + \frac{1.0}{s'} = \frac{1.0 - 1.59}{-4.0 \text{ cm}} = +\frac{0.59}{4.0 \text{ cm}} = 0.1475 \text{ cm}^{-1}$$

$$\Rightarrow \frac{1}{s'} = 0.1475 \text{ cm}^{-1} - 0.795 \text{ cm}^{-1} \Rightarrow s' = -1.54 \text{ cm}$$

That is, the bubble appears $1.54 \text{ cm} \approx 1.5 \text{ cm}$ beneath the surface.

23.36. **Model:** Assume the lens is thin.
Visualize:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{fs}{s - f}$$

Solve:

$$s' = \frac{fs}{s - f} = \frac{(20 \text{ cm})(60 \text{ cm})}{60 \text{ cm} - 20 \text{ cm}} = 30 \text{ cm}$$

The magnification is $m = -s'/s = -30 \text{ cm}/60 \text{ cm} = -1/2$. This means the image is inverted and has a height of 0.50 cm.

Assess: Ray tracing will confirm these results.

23.37. **Solve:** The image is at 40 cm as seen in the figure. It is inverted.
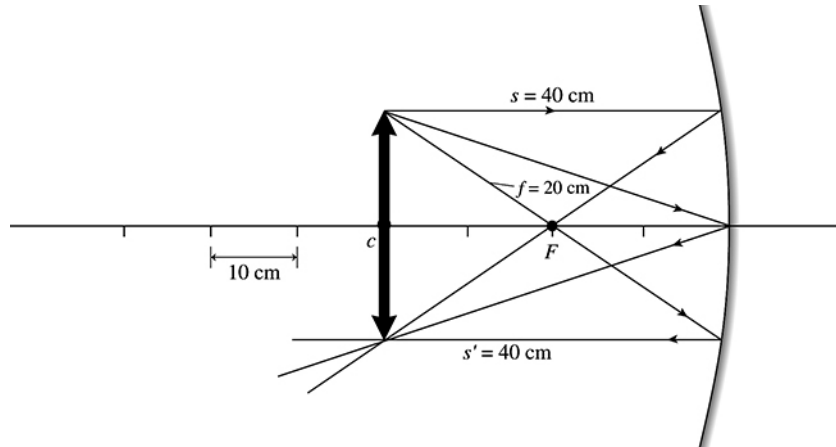
Assess: When the object is outside the focal length we get an inverted image.

23.38. **Solve:** The image is at -30 cm as seen in the figure. It is upright.
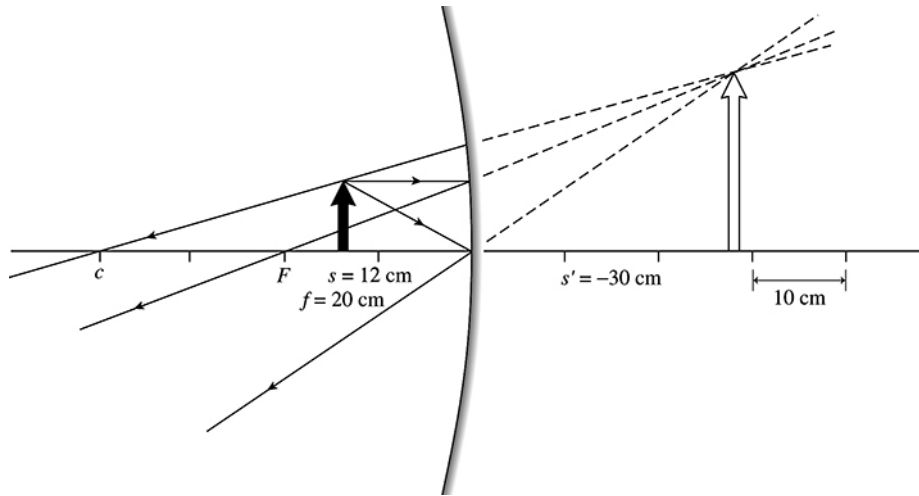
Assess: When the object is within the focal length we get a magnified upright image.

23.39. **Solve:** The image is at -12 cm as seen in the figure. It is upright.
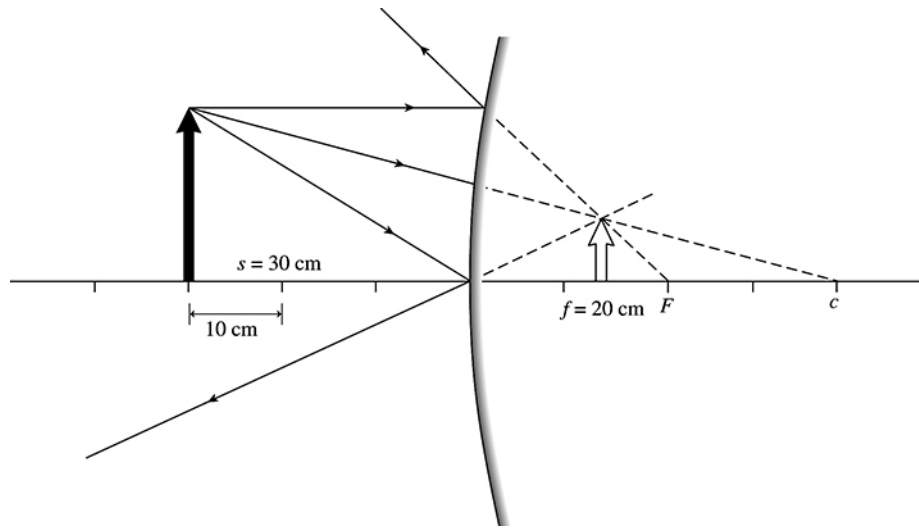
Assess: We expected an upright virtual image from the convex mirror.

23.40. **Model:** The speed of light in a material is determined by the refractive index as $v = c/n$.

Solve: To acquire data from memory, a total time of only 2.0 ns is allowed. This time includes 0.5 ns that the memory unit takes to process a request. Thus, the *travel time* for an infrared light pulse from the central processing unit to the memory unit and back is 1.5 ns. Let d be the distance between the central processing unit and the memory unit. The refractive index of silicon for infrared light is $n_{Si} = 3.5$. Then,

$$1.5 \text{ ns} = \frac{2d}{v_{Si}} = \frac{2d}{c/n_{Si}} = \frac{2dn_{Si}}{c} \Rightarrow d = \frac{(1.5 \text{ ns})c}{2n_{Si}} = \frac{(1.5 \times 10^{-9} \text{ s})(3.0 \times 10^{8} \text{ m/s})}{2(3.5)} \Rightarrow d = 6.4 \text{ cm}$$

23.41. **Model:** Treat the red ball as a point source and use the ray model of light.

Solve: **(a)** Using the law of reflection, we can obtain 3 images of the red ball.

(b) The images of the ball are located at B, C, and D. Relative to the intersection point of the two mirrors, the coordinates of B, C, and D are: B(+1 m, −2 m), C(−1 m, +2 m), and D(+1 m, +2 m).

(c)
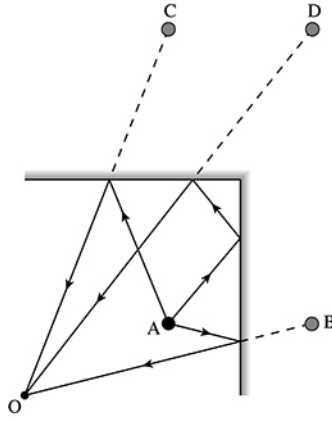
23.42. **Model:** Treat the laser beam as a ray and use the ray model of light.
Visualize:
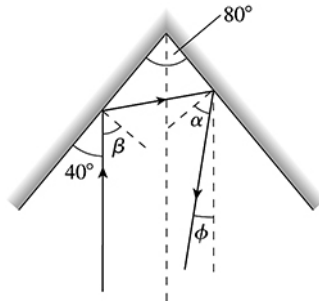
Solve: From the geometry of the mirrors and the rays, $\beta = 50°$, $\alpha = 30°$, and $\phi = 20°$.

23.43. **Model:** For a mirror, the image distance behind the mirror equals the object's distance in front of the mirror.

Visualize:
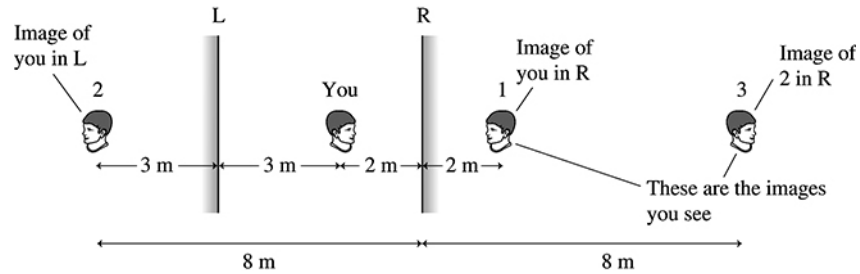
Solve: Your face is 2.0 from the mirror into which you are looking. The image of your face (image 1) is 2.0 m behind the mirror, or 4.0 m away. Behind you, the image of the back of your head (image 2) is 3.0 m behind the mirror on the other wall. You can't see this image because you're looking to the right. However, the reflected rays that appear to come from image 2 (a virtual image) act just like the rays from an object—that is, just as the rays would if the back of your head were really at the position of image 2. These rays reflect from the mirror 2.0 m in front of you into which you're staring and form an image (image 3) 8.0 m behind the mirror. This is the image of the back of your head that you see in the mirror in front of you. Since you're 2.0 m from the mirror, the image of the back of your head is 10 m away.

23.44. **Model:** Treat the laser beam as a ray and use the ray model of light.
Visualize:
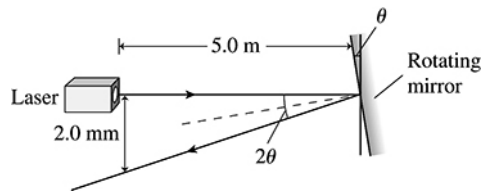
As the cylinder rotates by an angle θ, the path of the reflected laser beam changes by an angle 2θ relative to the direction of incidence.

Solve: Because the angle 2θ is very small,

$$\tan 2\theta \cong 2\theta = \frac{2.0 \times 10^{-3} \text{ m}}{5.0 \text{ m}} \Rightarrow \theta = \frac{1}{5000} \text{ rad} = \frac{180}{\pi(5000)} \text{ degrees} = 0.011°$$

23.45. **Model:** Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.
Visualize:
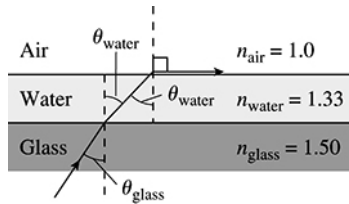
For angles θ_{water} that are less than the critical angle, light will be refracted into the air.
Solve: Snell's law at the water-air boundary is $n_{air} \sin\theta_{air} = n_{water} \sin\theta_{water}$. Because the maximum angle of θ_{air} is $90°$, we have

$$(1.0)\sin 90° = 1.33\sin\theta_{water} \Rightarrow \theta_{water} = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$$

Applying Snell's law again to the glass-water boundary,

$$n_{glass}\sin\theta_{glass} = n_{water}\sin\theta_{water} \Rightarrow \theta_{glass} = \sin^{-1}\left(\frac{n_{water}}{n_{glass}}\sin\theta_{water}\right) = \sin^{-1}\left(\frac{1.33(\sin 48.75°)}{1.50}\right) = 42°$$

Thus $42°$ is the maximum angle of incidence onto the glass for which the ray emerges into the air.

23.46. **Model:** Use the ray model of light.
Visualize:
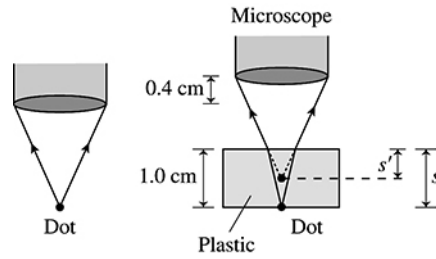
Solve: When the plastic is in place, the microscope focuses on the virtual image of the dot. From the figure, we note that $s = 1.0$ cm and $s' = 1.0$ cm $- 0.4$ cm $= 0.6$ cm. The rays are paraxial, and the object and image distances are measured relative to the plastic-air boundary. Using Equation 23.13,

$$s' = \frac{n_{air}}{n_{plastic}} s \Rightarrow 0.6 \text{ cm} = \frac{1.0}{n_{plastic}}(1.0 \text{ cm}) \Rightarrow n_{plastic} = \frac{1.0 \text{ cm}}{0.6 \text{ cm}} = 1.67$$

23.47. **Model:** Use the ray model of light and the law of refraction.

Solve: Snell's law at the air-glass boundary is $n_{air} \sin\theta_{air} = n_{glass} \sin\theta_{glass}$. We require $\theta_{glass} = \frac{1}{2}\theta_{air}$, so

$$n_{air} \sin\left(2\theta_{glass}\right) = n_{glass} \sin\theta_{glass} \Rightarrow n_{air}\left(2\sin\theta_{glass}\cos\theta_{glass}\right) = n_{glass}\sin\theta_{glass}$$

$$\Rightarrow \theta_{glass} = \cos^{-1}\left(\frac{n_{glass}}{2n_{air}}\right) = \cos^{-1}\left(\frac{1.50}{2\times1.0}\right) = 41.4° \Rightarrow \theta_{air} = 82.8°$$

23.48. **Model:** Use the ray model of light and the law of refraction.
Visualize:
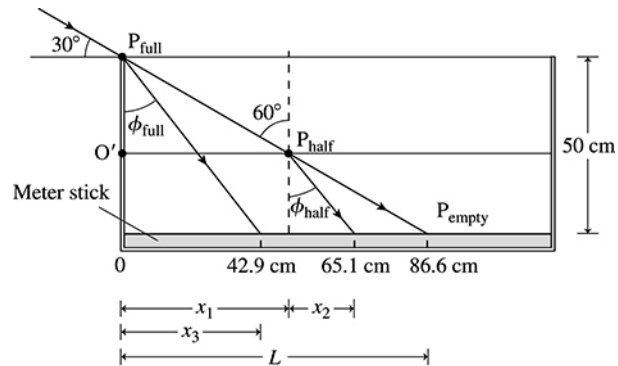
Solve: **(a)** The ray of light strikes the meter stick at P_{empty}, which is a distance L from the zero mark of the meter stick. So,

$$\tan 60° = \frac{L}{50 \text{ cm}} \Rightarrow L = (50 \text{ cm})\tan 60° = 86.6 \text{ cm}$$

(b) The ray of light refracts at P_{half} and strikes the meter stick a distance $x_1 + x_2$ from the zero of the meter stick. We can find x_1 from the triangle $P_{full}P_{half}O'$:

$$\tan 60° = \frac{x_1}{25 \text{ cm}} \Rightarrow x_1 = (25 \text{ cm})\tan 60° = 43.30 \text{ cm}$$

We also have $x_2 = (25 \text{ cm})\tan \phi_{half}$. Using Snell's law,

$$n_{air}\sin 60° = n_{water}\sin\phi_{half} \Rightarrow \phi_{half} = \sin^{-1}\left(\frac{\sin 60°}{1.33}\right) = 40.63°$$

$$\Rightarrow x_2 = (25 \text{ cm})\tan 40.63° = 21.45 \text{ cm} \Rightarrow x_1 + x_2 = 43.30 \text{ cm} + 21.45 \text{ cm} = 64.8 \text{ cm}$$

(c) The ray of light experiences refraction at P_{full} and the angle of refraction is the same as in part (b). We get

$$\tan\phi_{full} = \frac{x_3}{50 \text{ cm}} \Rightarrow x_3 = (50 \text{ cm})\tan 40.63° = 42.9 \text{ cm}$$

23.49. **Model:** Use the ray model of light. Light undergoes total internal reflection if it is incident on a boundary at an angle greater than the critical angle.

Visualize:
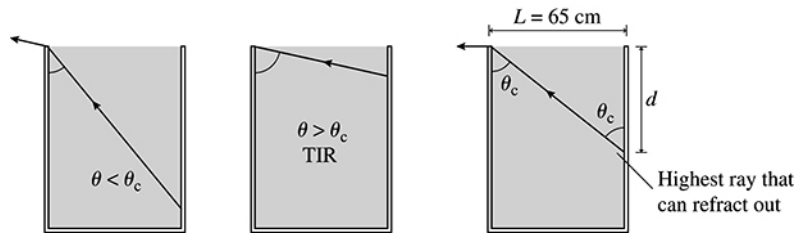
Solve: **(a)** To reach your eye, a light ray must refract through the *top* surface of the water and into the air. You can see in the figure that rays coming from the bottom of the tank are incident on the top surface at fairly small angles, but rays from the marks near the top of the tank are incident at very large angles—greater than the critical angle. These rays undergo total internal reflection in the water and do not exit into the air where they can be seen. Thus you can see the marks from the bottom of the tank upward.

(b) The highest point you can see is the one from which the ray reaches the top surface at the critical angle θ_c. For a water-air boundary, the critical angle is $\theta_c = \sin^{-1}(1/1.33) = 48.75°$. You can see from the figure that the depth of this point is such that

$$\frac{L}{d} = \tan\theta_c \Rightarrow d = \frac{L}{\tan\theta_c} = \frac{65.0 \text{ cm}}{\tan(48.75°)} = 57.0 \text{ cm}$$

Since the marks are every 10 cm, the high mark you can see is the one at 60 cm.

23.50. **Model:** Use the ray model of light and the law of refraction. Assume the sun is a point source of light.
Visualize:
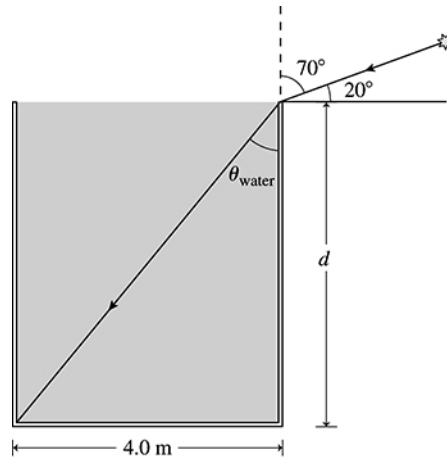
When the bottom of the pool becomes completely shaded, a ray of light that is incident at the top edge of the swimming pool does not reach the bottom of the pool after refraction.

Solve: The depth of the swimming pool is $d = 4.0 \text{ m}/\tan\theta_{\text{water}}$. We will find the angle by using Snell's law. We have

$$n_{\text{water}} \sin\theta_{\text{water}} = n_{\text{air}} \sin 70° \Rightarrow \theta_{\text{water}} = \sin^{-1}\left(\frac{\sin 70°}{1.33}\right) = 44.95° \Rightarrow d = \frac{4.0 \text{ m}}{\tan 44.95°} = 4.0 \text{ m}$$

23.51. **Model:** Use the ray model of light and the law of refraction. Assume that the laser beam is a ray of light.

Visualize:
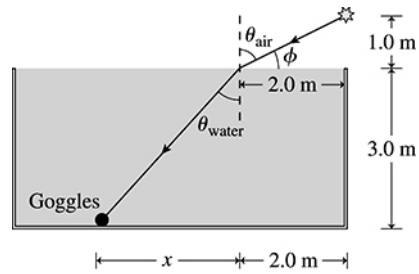
The laser beam enters the water 2.0 m from the edge, undergoes refraction, and illuminates the goggles. The ray of light from the goggles then retraces its path and enters your eyes.

Solve: From the geometry of the diagram,

$$\tan\phi = \frac{1.0 \text{ m}}{2.0 \text{ m}} \implies \phi = \tan^{-1}(0.50) = 26.57° \implies \theta_{air} = 90° - 26.57° = 63.43°$$

Snell's law at the air-water boundary is $n_{air}\sin\theta_{air} = n_{water}\sin\theta_{water}$. Using the above result,

$$(1.0)\sin 63.43° = 1.33\sin\theta_{water} \implies \theta_{water} = \sin^{-1}\left(\frac{\sin 63.43°}{1.33}\right) = 42.26°$$

Taking advantage of the geometry in the diagram again,

$$\frac{x}{3.0 \text{ m}} = \tan\theta_{water} \implies x = (3.0 \text{ m})\tan 42.26° = 2.73 \text{ m}$$

The distance of the goggles from the edge of the pool is 2.73 m + 2.0 m = 4.73 m ≈ 4.7 m.

23.52. **Model:** Use the ray model of light and the law of refraction. Assume that the laser beam is a ray of light.
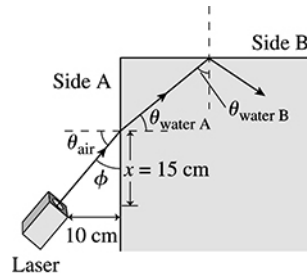**Visualize:**

Solve: **(a)** From the geometry of the diagram at side A,

$$\tan\phi = \frac{10\text{ cm}}{15\text{ cm}} \implies \phi = \tan^{-1}\left(\frac{10}{15}\right) \implies \phi = 33.69°$$

This means the angle of incidence at side A is $\theta_{\text{air}} = 90° - 33.69° = 56.31°$. Using Snell's law at side A,

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{water}}\sin\theta_{\text{water A}} \implies \theta_{\text{water A}} = \sin^{-1}\left(\frac{1.0\sin 56.31°}{1.330}\right) = 38.73°$$

This ray of light now strikes side B. The angle of incidence at this water-air boundary is $\theta_{\text{water B}} = 90° - \theta_{\text{water A}} = 51.3°$. The critical angle for the water-air boundary is

$$\theta_{\text{c}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.0}{1.33}\right) = 48.8°$$

Because the angle $\theta_{\text{water B}}$ is larger than θ_{c}, the ray will experience total internal reflection.
(b) We will now repeat the above calculation with $x = 25$ cm. From the geometry of the diagram at side A, $\phi = 21.80°$ and $\theta_{\text{air}} = 68.20°$. Using Snell's law at the air-water boundary, $\theta_{\text{water A}} = 44.28°$ and $\theta_{\text{water B}} = 45.72°$.
Because $\theta_{\text{water B}} < \theta_{\text{c}}$, the ray will be refracted into the air. The angle of refraction is calculated as follows:

$$n_{\text{air}}\sin\theta_{\text{air B}} = n_{\text{water}}\sin\theta_{\text{water B}} \implies \theta_{\text{air B}} = \sin^{-1}\left(\frac{1.33\sin 45.72°}{1}\right) = 72.2°$$

(c) Using the critical angle for the water-air boundary found in part (a), $\theta_{\text{water A}} = 90° - 48.75° = 41.25°$. According to Snell's law,

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{water}}\sin\theta_{\text{water A}} \implies \theta_{\text{air}} = \sin^{-1}\left(\frac{1.33\sin 41.25°}{1.0}\right) = 61.27°$$

$$\implies \phi = 90° - 61.27° = 28.73°$$

The minimum value of x for which the laser beam passes through side B and emerges into the air is calculated as follows:

$$\tan\phi = \frac{10\text{ cm}}{x} \implies x = \frac{10\text{ cm}}{\tan 28.73°} = 18.2\text{ cm} \approx 18\text{ cm}$$

23.53. **Model:** Use the ray model of light. Assume the bonfire is a point source right at the corner of the lake.

Visualize:
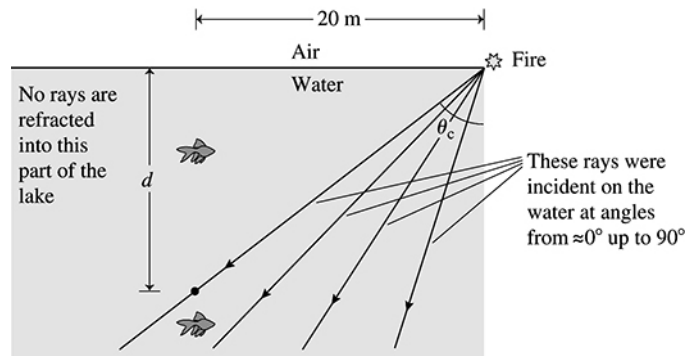
Solve: **(a)** Light rays from the fire enter the lake right at the edge. Even though the rays in air are incident on the surface at a range of angles from ≈0° up to 90°, the larger index of refraction of water causes the rays to travel downward in the water with angles $\leq\theta_c$, the critical angle. Some of these rays can reach a fish that is deep in the lake, but a shallow fish out from shore is in the "exclusion zone" that is not reached by any rays from the fire. Thus a fish needs to be deep to see the light from the fire.

(b) The shallowest fish that can see the fire is one that receives light rays refracting into the water at the critical angle θ_c. These are rays that were incident on the water's surface at ≈90°. The critical angle for a water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.33}\right) = 48.75°$$

The fish is 20 m from shore, so its depth is

$$d = \frac{20\ \text{m}}{\tan(48.75°)} = 17.5\ \text{m} \approx 18\ \text{m}$$

That is, a fish 20 m from shore must be at least 18 m deep to see the fire.

23.54. **Model:** Use the ray model of light. Assume that the target is a point source of light.
Visualize:
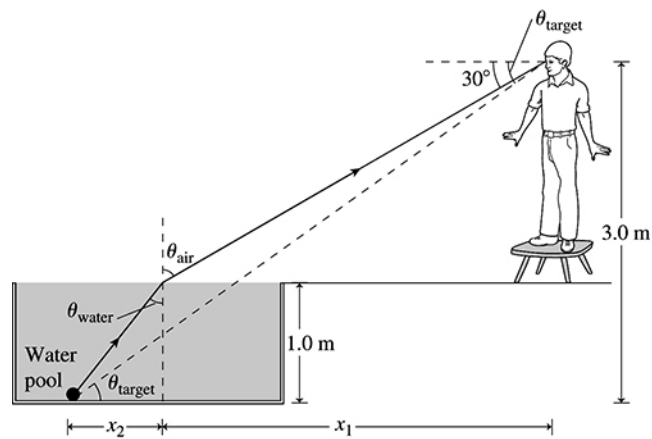
Solve: From the geometry of the figure with $\theta_{air} = 60°$,

$$\tan\theta_{air} = \frac{x_1}{2.0 \text{ m}} \Rightarrow x_1 = (2.0 \text{ m})(\tan 60°) = 3.464 \text{ m}$$

Let us find the horizontal distance x_2 by applying Snell's law to the air-water boundary. We have

$$n_{water} \sin\theta_{water} = n_{air} \sin\theta_{air} \Rightarrow \theta_{water} = \sin^{-1}\left(\frac{\sin 60°}{1.33}\right) = 40.63°$$

Using the geometry of the diagram,

$$\frac{x_2}{1.0 \text{ m}} = \tan\theta_{water} \Rightarrow x_2 = (1.0 \text{ m})\tan 40.63° = 0.858 \text{ m}$$

To determine θ_{target}, we note that

$$\tan\theta_{target} = \frac{3.0 \text{ m}}{x_1 + x_2} = \frac{3.0 \text{ m}}{3.464 \text{ m} + 0.858 \text{ m}} = 0.6941 \Rightarrow \theta_{target} = 35°$$

23.55. **Model:** Use the ray model of light and the phenomena of refraction and dispersion.
Visualize:
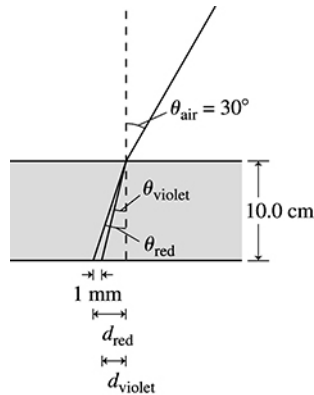
The refractive index of violet light is greater than the refractive index of red light. The violet wavelength thus gets refracted more than the red wavelength.
Solve: Using Snell's law for the red light at the air-glass boundary,

$$n_{air} \sin\theta_{air} = n_{red} \sin\theta_{red} \Rightarrow \theta_{red} = \sin^{-1}\left(\frac{n_{air} \sin\theta_{air}}{n_{red}}\right) = \sin^{-1}\left(\frac{1.0\sin 30°}{1.513}\right) = 19.30°$$

From the geometry of the diagram,

$$\frac{d_{red}}{10.0 \text{ cm}} = \tan\theta_{red} \qquad \frac{d_{violet}}{10.0 \text{ cm}} = \tan\theta_{violet}$$

$$\Rightarrow d_{red} = (10.0 \text{ cm})\tan(19.30°) = 3.502 \text{ cm} \Rightarrow d_{violet} = 3.502 \text{ cm} - 0.1 \text{ cm} = 3.402 \text{ cm}$$

$$\Rightarrow \theta_{violet} = \tan^{-1}\left(\frac{d_{violet}}{10.0 \text{ cm}}\right) = \tan^{-1}\left(\frac{3.402 \text{ cm}}{10.0 \text{ cm}}\right) = 18.79°$$

That is, white light is incident on a piece of glass at 30°, and it gets dispersed. The violet light makes an angle of 18.79° with the vertical. Using Snell's law,

$$n_{violet} \sin\theta_{violet} = n_{air} \sin\theta_{air} \Rightarrow n_{violet} = \frac{(1.0)\sin 30°}{\sin 18.79°} = 1.552$$

23.56. **Model:** Use the ray model of light and the phenomena of refraction and dispersion.
Visualize:
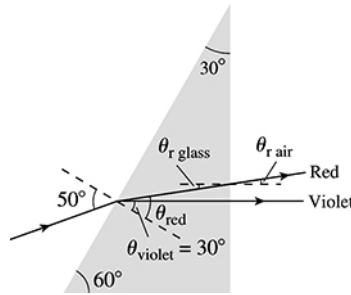
Solve: Since violet light is perpendicular to the second surface, it must reflect at $\theta_{violet} = 30°$ at the first surface. Using Snell's law at the air-glass boundary where the ray is incident,

$$n_{air} \sin\theta_{air} = n_{violet} \sin\theta_{violet} \Rightarrow n_{violet} = \frac{n_{air} \sin\theta_{air}}{\sin\theta_{violet}} = \frac{(1.0)\sin 50°}{\sin 30°} = 1.5321$$

Since $n_{violet} = 1.02\, n_{red}$, $n_{red} = 1.5021$. Using Snell's law for the red light at the first surface

$$n_{red} \sin\theta_{red} = n_{air} \sin\theta_{air} \Rightarrow \theta_{red} = \sin^{-1}\left(\frac{1.0\sin 50°}{1.5021}\right) = 30.664°$$

The angle of incidence on the rear face of the prism is thus $\theta_{r\,glass} = 30.664° - 30° = 0.664°$. Using Snell's law once again for the rear face and for the red wavelength,

$$n_{red} \sin\theta_{r\,glass} = n_{air} \sin\theta_{r\,air} \Rightarrow \theta_{r\,air} = \sin^{-1}\left(\frac{n_{red} \sin\theta_{r\,glass}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.5021\sin 0.664°}{1.0}\right) = 0.997°$$

Because $\theta_{v\,air} = 0°$ and $\theta_{r\,air} = 0.997°$, $\phi = \theta_{r\,air} - \theta_{v\,air} = 0.997° \cong 1.00°$.

23.57. **Model:** Use the ray model of light and the phenomenon of refraction.

Visualize:
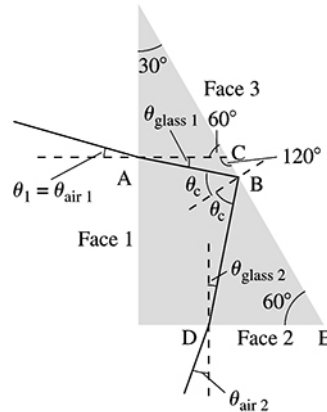
Solve: **(a)** The critical angle θ_c for the glass-air boundary is

$$n_{glass}\sin\theta_c = n_{air}\sin 90° \Rightarrow \theta_c = \sin^{-1}\left(\frac{1.0}{1.50}\right) = 41.81°$$

For the triangle ABC,

$$\theta_{glass\ 1} + 120° + \left(90° - \theta_c\right) = 180° \Rightarrow \theta_{glass\ 1} = 180° - 120° - \left(90° - 41.81°\right) = 11.81°$$

Having determined $\theta_{glass\ 1}$, we can now find $\theta_{air\ 1}$ by using Snell's law:

$$n_{air}\sin\theta_{air\ 1} = n_{glass}\sin\theta_{glass\ 1} \Rightarrow \theta_{air\ 1} = \sin^{-1}\left(\frac{1.50\times\sin 11.81°}{1.0}\right) = 17.88°$$

Thus, the smallest angle θ_1 for which a laser beam will undergo TIR on the hypotenuse of this glass prism is 17.9°.

(b) After reflecting from the hypotenuse (face 3) the ray of light strikes the base (face 2) and refracts into the air. From the triangle BDE,

$$\left(90° - \theta_{glass\ 2}\right) + 60° + \left(90° - \theta_c\right) = 180° \Rightarrow \theta_{glass\ 2} = 90° + 60° + 90° - 41.81° - 180° = 18.19°$$

Snell's law at the glass-air boundary of face 2 is

$$n_{glass}\sin\theta_{glass\ 2} = n_{air}\sin\theta_{air\ 2} \Rightarrow \theta_{air\ 2} = \sin^{-1}\left(\frac{n_{glass}\sin\theta_{glass\ 2}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.50\sin 18.19°}{1.0}\right) = 27.9°$$

Thus the ray exits 27.9° left of the normal.

23.58. Model: Use the ray model of light.

Solve: **(a)** Using Snell's law at the air-glass boundary, with ϕ being the angle of refraction inside the prism,

$$n_{\text{air}} \sin \beta = n \sin \phi \implies \sin \beta = n \sin \phi$$

Considering the triangle made by the apex angle and the refracted ray,

$$(90° - \phi) + (90° - \phi) + \alpha = 180° \implies \phi = \tfrac{1}{2}\alpha$$

Thus

$$\sin \beta = n \sin \left(\tfrac{1}{2}\alpha\right) \implies \beta = \sin^{-1}\left(n \sin(\tfrac{1}{2})\alpha\right)$$

(b) Using the above expression, we obtain

$$n = \frac{\sin \beta}{\sin(\tfrac{1}{2}\alpha)} = \frac{\sin 52.2°}{\sin 30°} = 1.58$$

23.59. **Model:** The bubble is a point source of light. The surface is a spherical refracting surface.
Solve: The bubble is in zircon, so $n_1 = 1.96$ and $n_2 = 1.00$. The surface is concave (object facing into a "cave") as seen from the bubble (the object), so $R = -3.0$ cm. Equation 23.21 is

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.96}{3.0 \text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.96}{-3.0 \text{ cm}} \Rightarrow s' = -3.0 \text{ cm}$$

Thus, seen from outside, the bubble appears to be 3.0 cm beneath the surface. That is, a bubble at the center actually appears to be at the center.

23.60. **Model:** Use the ray model of light. The surface is a spherically refracting surface.
Visualize:
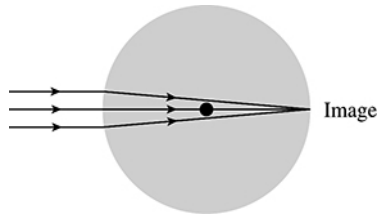
Solve: Because the rays are parallel, $s = \infty$. The rays come to focus on the rear surface of the sphere, so $s' = 2R$, where R is the radius of curvature of the sphere. Using Equation 23.21,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{\infty} + \frac{n}{2R} = \frac{n-1}{R} \Rightarrow n = 2.00$$

23.61. **Model:** Assume that the converging lens is a thin lens. Use ray tracing to locate the image.
Solve: **(a)**
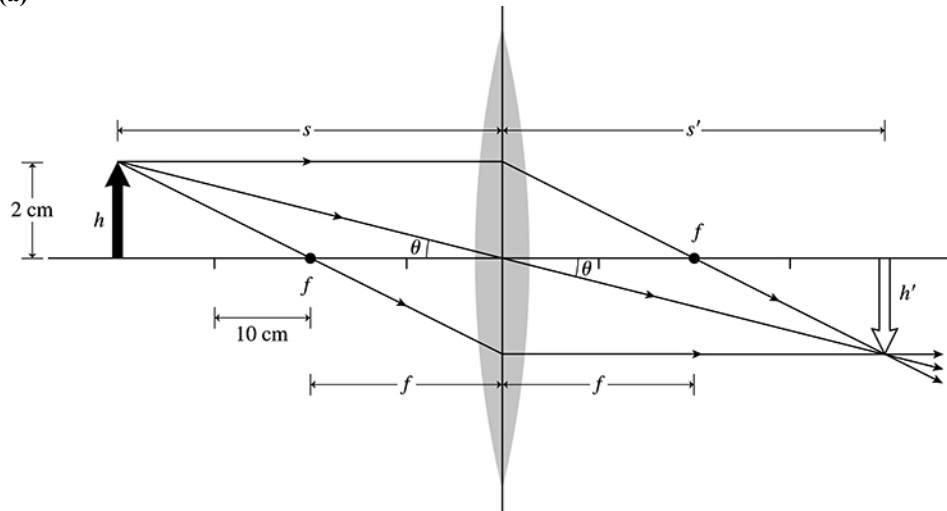
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. The three rays after refraction converge to give an image at $s' = 40$ cm. The height of the image is $h' = 2$ cm.
(b) Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{40 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}} \Rightarrow \frac{1}{s'} = \frac{1}{40 \text{ cm}} \Rightarrow s' = 40 \text{ cm}$$

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{40 \text{ cm}}{40 \text{ cm}} = -1$$

The image is inverted and as tall as the object, that is, $h' = 2.0$ cm. The values for h' and s' obtained in parts (a) and (b) agree.

23.62. **Model:** Use ray tracing to locate the image. Assume that the converging lens is a thin lens.
Solve: **(a)**

The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. The three special rays that experience refraction do not converge at a point. Instead they appear to come from a point that is 15 cm on the same side as the object itself. Thus $s' = -15$ cm. The image is upright and has a height of $h' = 1.5$ cm.
(b) Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{10 \text{ cm}} + \frac{1}{s'} = \frac{1}{30 \text{ cm}} \Rightarrow \frac{1}{s'} = -\frac{1}{15 \text{ m}} \Rightarrow s' = -15 \text{ cm}$$

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{-15 \text{ cm}}{10 \text{ cm}} = +1.5$$

The image is upright and 1.5 times the object, that is, 1.5 cm high. These values agree with those obtained in part (a).

23.63. **Model:** Use ray tracing to locate the image. Assume that the converging lens is a thin lens.
Solve: **(a)**

The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. The three special rays after refracting do not converge. Instead the rays appear to come from a point that is 60 cm on the same side of the lens as the object, so $s' = -60$ cm. The image is upright and has a height of 8.0 cm.
(b) Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{15 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}} \Rightarrow \frac{1}{s'} = -\frac{1}{60 \text{ cm}} \Rightarrow s' = -60 \text{ cm}$$

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{-60 \text{ cm}}{15 \text{ cm}} = +4$$

Thus, the image is 4 times larger than the object or $h' = Mh = 4h = 4(2.0 \text{ cm}) = 8.0$ cm. The image is upright. These values agree with those obtained in part (a).

23.64. **Model:** Use ray tracing to locate the image. Assume the converging lens is a thin lens.

Solve: **(a)**

The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. After refraction, the three special rays converge and give an image 50 cm away from the converging lens. Thus, $s' = +50$ cm. The image is inverted and its height is 0.65 cm.

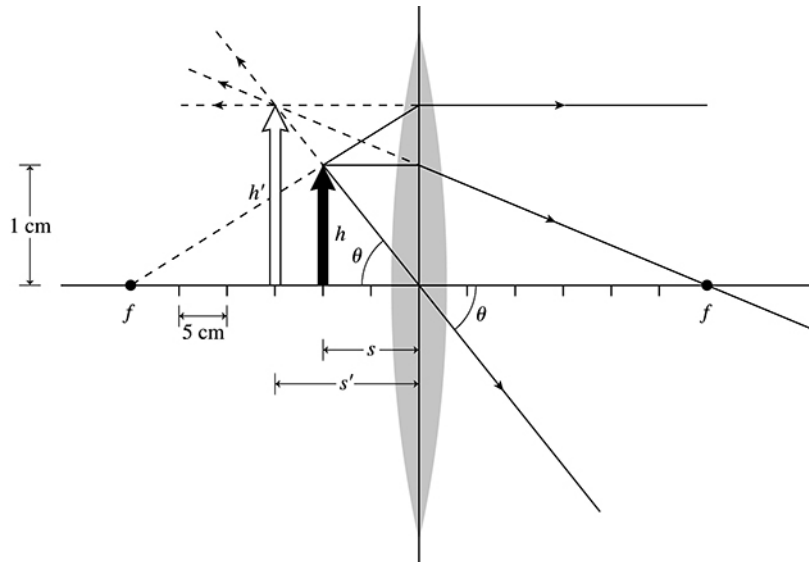**(b)** Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{75 \text{ cm}} + \frac{1}{s'} = \frac{1}{30 \text{ cm}} \Rightarrow \frac{1}{s'} = \frac{1}{50 \text{ cm}} \Rightarrow s' = 50 \text{ cm}$$

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{50 \text{ cm}}{75 \text{ cm}} = -\frac{2}{3}$$

The image height is $h' = Mh = (-2/3)(1 \text{ cm}) = -0.67$ cm. Because of the negative sign, the image is inverted. These results agree with those obtained in part (a).

23.65. **Model:** Use ray tracing to locate the image. Assume the diverging lens is a thin lens.
Solve: **(a)**
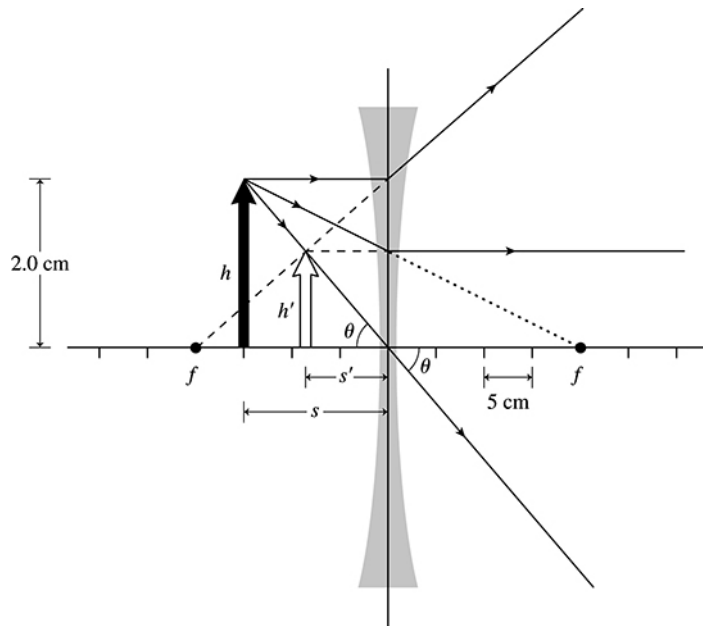
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.3. After refraction, the three special rays do not converge. The rays, on the other hand, appear to meet at a point that is 8.5 cm on the same side of the lens as the object. So $s' = -8.5$ cm. The image is upright and has a height of 1.1 cm.
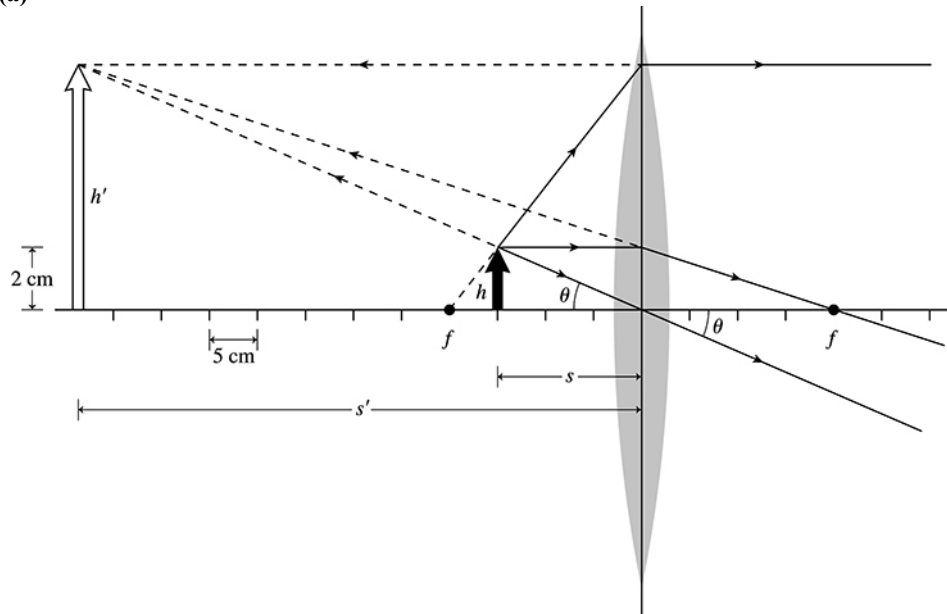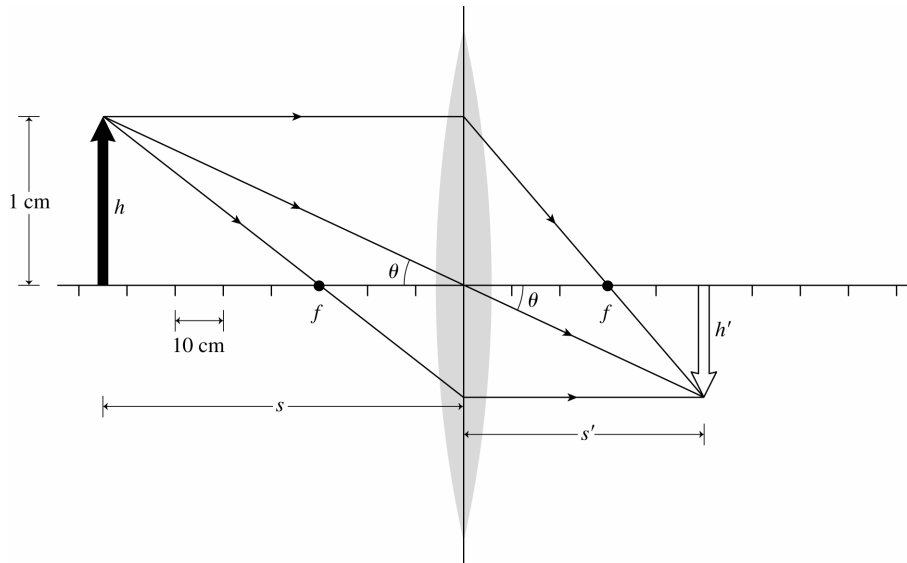**(b)** Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{15 \text{ cm}} + \frac{1}{s'} = \frac{1}{-20 \text{ cm}} \quad \Rightarrow \frac{1}{s'} = -\frac{7}{60 \text{ cm}} \Rightarrow s' = -\frac{60}{7} \text{ cm} = -8.6 \text{ cm}$$

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{\left(-60/7 \text{ cm}\right)}{15 \text{ cm}} = +\frac{4}{7} = 0.57$$

Thus, the image is 0.57 times larger than the object, or $h' = Mh = (0.57)(2.0 \text{ cm}) = 1.14$ cm. The image is upright because M is positive. These values agree, within measurement accuracy, with those obtained in part (a).

23.66. **Model:** Use ray tracing to locate the image. Assume the diverging lens is a thin lens.
Solve: **(a)**
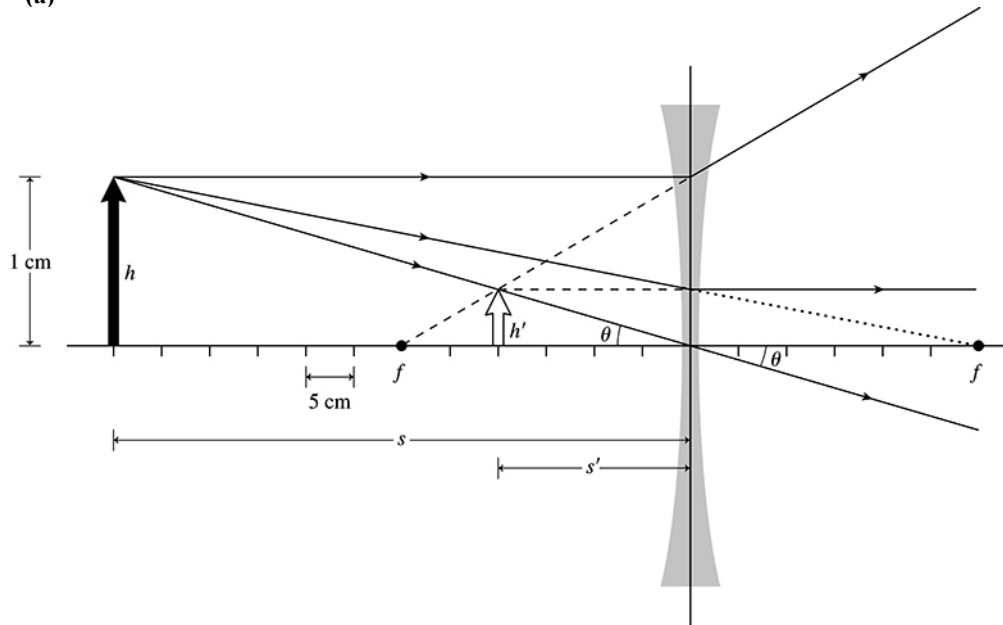
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.3. After refraction from the diverging lens, the three special rays do not converge. However, the rays appear to meet at a point that is 20 cm on the same side as the object. So $s' = -20$ cm. The image is upright and has a height of 0.3 cm.
(b) Using the thin-lens formula,

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-30 \text{ cm}} - \frac{1}{60 \text{ cm}} = -\frac{1}{20 \text{ cm}} \Rightarrow s' = -20 \text{ cm}$$

The image height is obtained from

$$M = -\frac{s'}{s} = -\frac{-20 \text{ cm}}{60 \text{ cm}} = \frac{1}{3} = 0.33$$

Thus, $h' = Mh = (0.33)(1.0 \text{ cm}) = 0.33$ cm, and the image is upright because M is positive. These values for s' and h' agree with those obtained in part (a).

23.67. **Visualize:**   Refer to Figure 23.57.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{fs}{s-f}$$

We are given $f = 60$ cm, $s = 20$ cm, and $h = 1.0$ cm.

Solve:

$$s' = \frac{fs}{s-f} = \frac{(60 \text{ cm})(20 \text{ cm})}{20 \text{ cm} - 60 \text{ cm}} = -30 \text{ cm}$$

The negative sign means the image is behind the mirror; it is a virtual image. The magnification is $m = -s'/s = 30 \text{ cm}/20 \text{ cm} = 1.5$.   This   means   the   image   is   upright   and   has   a   height   of $h' = mh = (1.5)(1.0 \text{ cm}) = 1.5 \text{ cm}$.

Assess: Ray tracing will confirm these results.

23.68. **Visualize:** Refer to Figure 23.55.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{fs}{s-f}$$

We are given $f = -60$ cm, $s = 20$ cm, and $h = 1.0$ cm.

Solve:

$$s' = \frac{fs}{s-f} = \frac{(-60 \text{ cm})(20 \text{ cm})}{20 \text{ cm} + 60 \text{ cm}} = -15 \text{ cm}$$

The negative sign means the image is behind the mirror; it is a virtual image. The magnification is $m = -s'/s = 15 \text{ cm}/20 \text{ cm} = 0.75$. This means the image is upright and has a height of $h' = mh = (0.75)(1.0 \text{ cm}) = 0.75 \text{ cm}$.

Assess: Ray tracing will confirm these results.

23.69. **Model:** Assume the lens is a thin lens and the thin-lens formula applies.

Solve: Because we want to form an image of the spider on the wall, the image is real and we need a converging lens. That is, both s' and s are positive. This also implies that the spider's image is inverted, so $M = -s'/s = -\frac{1}{2}$.

Using the thin-lens formula with $s' = \frac{1}{2}s$,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{\frac{1}{2}s} = \frac{1}{f} \Rightarrow \frac{3}{s} = \frac{1}{f} \Rightarrow f = \frac{s}{3}$$

We also know that the spider is 2.0 m from the wall, so

$$s + s' = 2.0 \text{ m} = s + \tfrac{1}{2}s \Rightarrow s = \tfrac{1}{3}(4.0 \text{ m}) = 133.3 \text{ cm}$$

Thus, $f = \frac{1}{3}s = 44$ cm and $s' = 2.0 \text{ m} - 1.33 \text{ m} = 0.67 \text{ m} = 67$ cm. We need a 44 cm focal length lens placed 67 cm from the wall.

23.70. **Model:** Assume the lens to be a thin lens.

Solve: Because we want to form an image of the candle on the wall, we need a converging lens. We have $s + s' = 200$ cm. Using the thin-lens formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{200 \text{ cm} - s} = \frac{1}{32 \text{ cm}} \Rightarrow s^2 - (200 \text{ cm})s + 6400 \text{ cm}^2 = 0$$

The two solutions to this equation are $s = 160$ cm and 40 cm. When $s = 160$ cm, then $s' = 200$ cm $- 160$ cm $= 40$ cm. The magnification is

$$M = -\frac{s'}{s} = -\frac{40 \text{ cm}}{160 \text{ cm}} = -0.25$$

so the image is inverted and its height is (2.0 cm)(0.25) = 0.50 cm. When $s = 40$ cm, then $s' = 200$ cm $- 40$ cm $= 160$ cm. The magnification is

$$M = -\frac{s'}{s} = -\frac{160 \text{ cm}}{40 \text{ cm}} = -4$$

so the image is again inverted and its height is (2.0 cm)(4) = 8.0 cm.

23.71. **Model:** The eye is a converging lens and assume it is a thin lens.

Solve: **(a)** The diameter of an adult eyeball is typically 4.0 cm.

(b) The near point distance is approximately 10 inches ≈ 25 cm.

(c) Using the thin-lens formula,

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{25\text{ cm}}+\frac{1}{4.0\text{ cm}}=\frac{1}{f} \Rightarrow \frac{1}{f}=\frac{29}{100\text{ cm}} \Rightarrow f\approx 3.4\text{ cm}$$

23.72. **Model:** Assume the projector lens is a thin lens.
Solve: **(a)** The absolute value of the magnification of the lens is

$$|M| = \left|\frac{h'}{h}\right| = \left|\frac{98 \text{ cm}}{2 \text{ cm}}\right| = 49$$

Because the projector forms a real image of a real object, the image will be inverted. Thus,

$$M = -49 = -\frac{s'}{s} \Rightarrow s' = 49s$$

We also have

$$s + s' = 300 \text{ cm} \Rightarrow s + 49s = 300 \text{ cm} \Rightarrow s = 6.0 \text{ cm} \Rightarrow s' = 294 \text{ cm}$$

Using these values of s and s', we can find the focal length of the lens:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{6.0 \text{ cm}} + \frac{1}{294 \text{ cm}} \Rightarrow f = 5.9 \text{ cm}$$

(b) From part (a) the lens should be 6.0 cm from the slide.

23.73. **Visualize:** The lens must be a converging lens for this scenario to happen, so we expect f to be positive. In the first case the upright image is virtual $(s'_2 < 0)$ and the object must be closer to the lens than the focal point. The lens is then moved backward past the focal point and the image becomes real $(s'_2 > 0)$.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = \frac{ss'}{s + s'}$$

We are given $s_1 = 10\,\text{cm}$ and $m_1 = 2$.

Solve: Since the first image is virtual, $s' < 0$. We are told the first magnification is $m_1 = 2 = -s'_1/s_1 \Rightarrow s'_1 = -20$ cm. We can now find the focal length of the lens.

$$f = \frac{s_1 s'_1}{s_1 + s'_1} = \frac{(10\text{ cm})(-20\text{ cm})}{10\text{ cm} - 20\text{ cm}} = 20\text{ cm}$$

After the lens is moved, $m_2 = -2 = -s'_2/s_2$. Start with the thin lens equation again.

$$\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f}$$

Replace s'_2 with $-m_2 s_2$.

$$\frac{1}{s_2} + \frac{1}{-m_2 s_2} = \frac{1}{f}$$

Now solve for s_2.

$$\frac{-m_2 s_2 + s_2}{s_2(-m_2 s_2)} = \frac{1}{f}$$

$$\frac{-m_2 s_2 + s_2}{-m_2 s_2^2} = \frac{1}{f}$$

Cancel one s_2.

$$\frac{m_2 - 1}{m_2 s_2} = \frac{1}{f}$$

Multiply both sides by $f s_2$.

$$s_2 = f\left(\frac{m_2 - 1}{m_2}\right) = (20\text{ cm})\left(\frac{-2 - 1}{-2}\right) = 30\text{ cm}$$

The distance the lens moved is $s_2 - s_1 = 30\text{ cm} - 10\text{ cm} = 20\text{ cm}$.

Assess: We knew s_2 needed to be bigger than f; it is, and is in a reasonable range. The final answer for the distance the lens moved also seems reasonable.

23.74. **Model:** Assume the symmetric converging lens is a thin lens.

Solve: Because the lens forms a real image on the screen of a real object, the image is inverted. Thus, $M = -2 = -s'/s$. Also,

$$s + s' = 60 \text{ cm} \implies s + 2s = 60 \text{ cm} \implies s = 20 \text{ cm} \implies s' = 40 \text{ cm}$$

We can use the thin-lens formula to determine the radius of curvature of the symmetric converging lens $(|R_1| = |R_2|)$ as follows:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Using $R_1 = +|R|$ (convex toward the object), $R_2 = -|R|$ (concave toward the object), and $n = 1.59$,

$$\frac{1}{20 \text{ cm}} + \frac{1}{40 \text{ cm}} = (1.59 - 1)\left(\frac{1}{|R|} - \frac{1}{-|R|}\right) \implies \frac{3}{40 \text{ cm}} = \frac{1.18}{|R|} \implies |R| = 15.7 \text{ cm} \approx 16 \text{ cm}$$

23.75. **Visualize:** We are given $f = R/2 = 40 \text{ cm}/2 = 20 \text{ cm}$. We are also given $m = -s'/s = 3$.

Solve: Solve the thin lens equation for s'.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$s' = \frac{fs}{s - f}$$

Plug this into the magnification equation, $s = -s'/m$.

$$s = -\frac{1}{m}\frac{fs}{s - f}$$

Cancel an s from the numerator of each side, and multiply both sides by $s - f$.

$$s - f = -\frac{f}{m}$$

$$s = f - \frac{f}{m} = f\left(1 - \frac{1}{m}\right) = (20 \text{ cm})\left(1 - \frac{1}{3}\right) = 13.3 \text{ cm}$$

Assess: This answer is within the focal length of the concave mirror as we expect for an upright, magnified, virtual image.

23.76. **Visualize:** We are given $h = 2.0$ cm and $h' = 1.0$ cm so we know $m = 0.5$.
Solve: Solve $m = -s'/s$ for s.

$$s = \frac{-s'}{m}$$

Plug this result into the thin lens equation.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = \frac{ss'}{s+s'}$$

$$f = \frac{ss'}{s+s'} = \frac{\left(\dfrac{-s'}{m}\right)(s')}{\dfrac{-s'}{m} + s'} = \frac{\left(\dfrac{-s'}{m}\right)(s')}{s'\left(1 - \dfrac{1}{m}\right)}$$

Cancel one s' and distribute the m in the denominator.

$$f = \frac{-s'}{m\left(1 - \dfrac{1}{m}\right)} = \frac{-s'}{m-1} = \frac{150 \text{ cm}}{0.5 - 1} = -300 \text{ cm}$$

Assess: The negative value for f tells us this is a convex mirror.

23.77. **Visualize:** We are given $f = 25$ cm. From the thin mirror equation we know that $s' = \dfrac{fs}{s-f}$. The five

points under consideration are $s = 50$ cm, 75 cm, 100 cm, 125 cm, 150 cm.

Solve: (a) Use $s' = \dfrac{fs}{s-f}$ for each of the five points.

s (cm)	s' (cm)	h (cm)	h' (cm)
50	50	10	10
75	37.5	10	5.0
100	33.3	10	3.3
125	31.3	10	2.5
150	30	10	2.0

(b)
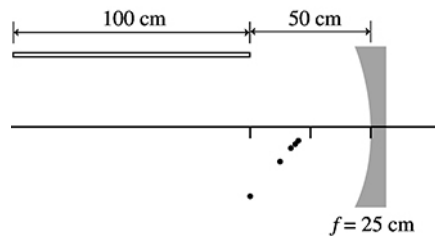
(c) The image is curved and not parallel to the axis.

Assess: For further reading, see "Longitudinal Magnification Drawing Mistake" by Héctor Rabal, Nelly Cam, and Marcelo Trivi in *The Physics Teacher*, vol. 42, January 2004, pp. 31–33, but know that Equation 4 there is missing a couple of minus signs.

23.78. **Visualize:** First concentrate on the optic axis and the ray parallel to it. Geometry says if parallel lines are both cut by a diagonal (in this case the line through the center of curvature and normal to the mirror at the point of incidence) the interior angles are equal; so $\phi = \theta_i$. The law of reflection says that $\theta_i = \theta_r$, so we conclude $\phi = \theta_r$. Now concentrate on the triangle whose sides are R, a, and b. Because two of the angles are equal then it is isosceles; therefore $b = a$. Apply the law of cosines to this triangle.
Solve:

$$b^2 = a^2 + R^2 - 2aR\cos\phi$$

Because $a = b$, they drop out.

$$R^2 = 2aR\cos\phi$$

$$R = 2a\cos\phi$$

We want to know how big a is in terms of R, so solve for a.

$$a = \frac{R}{2\cos\phi}$$

If $\phi \ll 1$ then $\cos\phi \approx 1$, so in the limit of small ϕ, $a = R/2$, and then since $f = R - a$ it must also be that

$$f = \frac{R}{2}$$

Assess: Many textbooks forget to stress that $f = R/2$ only in the limit of small ϕ, *i.e.*, for paraxial rays.

23.79. **Model:** Use the ray model of light and assume the lens is a thin lens.

Visualize: Please refer to Figure 23.47.

Solve: Let n_1 be the refractive index of the fluid and n_2 the refractive index of the lens. The lens consists of two spherical surfaces having radii of curvature R_1 and R_2 and the lens thickness $t \to 0$. For the refraction from the surface with radius R_1, we use Equation 23.21:

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{R_1}$$

For the refraction from surface with radius R_2,

$$\frac{n_2}{-s_1'} + \frac{n_1}{s_2'} = \frac{n_1 - n_2}{R_2}$$

A negative sign is used with s_1' because the image formed by the first surface of the lens is a virtual image. This virtual image is the object for the second surface. Adding the two equations,

$$\frac{n_1}{s_1} + \frac{n_1}{s_2'} = n_2 - n_1 \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \Rightarrow \frac{1}{s_1} + \frac{1}{s_2'} = \frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

(b) In air, R_1 = +40 cm (convex toward the object), R_2 = −40 cm (concave toward the object), n_1 = 1.0, and n_2 = 1.50. So,

$$\frac{1}{f} = \left(\frac{1.50 - 1.0}{1.0} \right) \left(\frac{1}{40 \text{ cm}} - \frac{1}{-40 \text{ cm}} \right) \Rightarrow f = 40 \text{ cm}$$

In water, n_1 = 1.33 and n_2 = 1.50. So,

$$\frac{1}{f} = \left(\frac{1.50 - 1.33}{1.33} \right) \left(\frac{1}{40 \text{ cm}} - \frac{1}{-40 \text{ cm}} \right) \Rightarrow f = 156 \text{ cm}$$

23.80. **Model:** Use the ray model of light.

Solve: **(a)** The time (t) is the time to travel from A to the interface (t_1) and from the interface to B (t_2). That is,

$$t = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1}{c/n_1} + \frac{d_2}{c/n_2} = \frac{n_1 d_1}{c} + \frac{n_2 d_2}{c} = \frac{n_1}{c}\sqrt{x^2 + a^2} + \frac{n_2}{c}\sqrt{(w-x)^2 + b^2}$$

(b) Because t depends on x and there is only one value of x for which the light travels from A to B in the least possible amount of time, we have

$$\frac{dt}{dx} = 0 = \frac{n_1 x}{c\sqrt{x^2 + a^2}} - \frac{n_2(w-x)}{c\sqrt{(w-x)^2 + b^2}}$$

The solution (hard to do!) would give x_{min}.
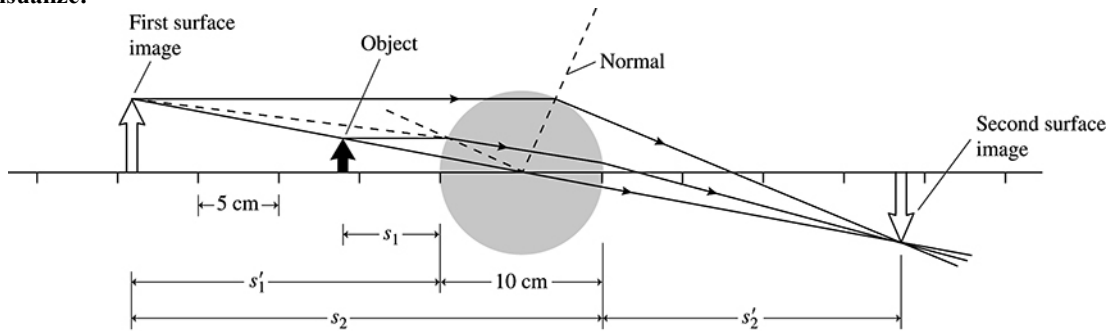
(c) From the geometry of the figure,

$$\frac{x}{\sqrt{x^2 + a^2}} = \frac{x}{d_1} = \sin\theta_1 \qquad \frac{w-x}{\sqrt{(w-x)^2 + b^2}} = \frac{w-x}{d_2} = \sin\theta_2$$

Thus, the condition of part (b) becomes

$$\frac{n_1}{c}\sin\theta_1 - \frac{n_2}{c}\sin\theta_2 = 0 \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

23.81. **Model:** Assume the ray model of light. The ball is *not* a thin lens. However, the image due to refraction from the first surface is the object for the second surface.

Visualize:

Solve: **(a)** For refraction from the first surface, $R = +5$ cm (convex toward the object). Thus,

$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.0}{6 \text{ cm}} + \frac{1.50}{s_1'} = \frac{0.50}{5 \text{ cm}} \Rightarrow \frac{1.50}{s_1'} = -\frac{1}{15 \text{ cm}} \Rightarrow s_1' = -22.5 \text{ cm}$$

The image is virtual (to the left of the surface) and upright. For refraction from the second surface, $s_2 = 22.5$ cm + 10 cm = 32.5 cm and $R = -5.0$ cm (concave toward the object). Thus,

$$\frac{1.50}{32.5 \text{ cm}} + \frac{1.0}{s_2'} = \frac{1-1.50}{-5.0 \text{ cm}} = \frac{1}{10 \text{ cm}} \Rightarrow \frac{1}{s_2'} = \frac{1}{10 \text{ cm}} - \frac{1.50}{32.5 \text{ cm}} \Rightarrow s_2' = 18.6 \text{ cm}$$
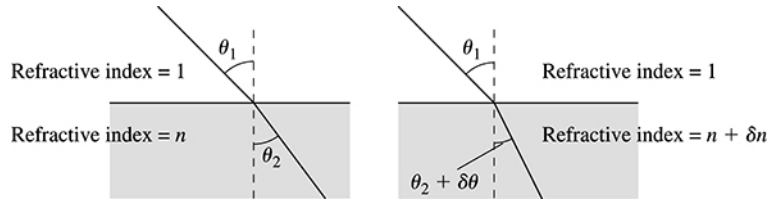
The image is 18.6 cm from the right edge of the ball and thus 23.6 cm from the center.
(b) The ray diagram showing the formation of the image is shown above.
(c) Using the thin-lens equation,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{6 \text{ cm} + 5 \text{ cm}} + \frac{1}{18.6 \text{ cm} + 5 \text{ cm}} = \frac{1}{f} \Rightarrow f = 7.5 \text{ cm}$$

23.82. **Model:** Use the ray model of light.
Visualize:

The angle of refraction is $\theta_2 + \delta\theta$ for those wavelengths that have a refractive index of $n + \delta n$.

Solve: **(a)** Applying Snell's law to the diagrams,

$$(1)\sin\theta_1 = n\sin\theta_2 \qquad (1)\sin\theta_1 = (n+\delta n)\sin(\theta_2 + \delta\theta)$$

Equating the right hand sides of the above two equations and using the formula for the sine of a sum,

$$n\sin\theta_2 = (n+\delta n)(\sin\theta_2 \cos\delta\theta + \cos\theta_2 \sin\delta\theta) = (n+\delta n)(\sin\theta_2 + \cos\theta_2 \delta\theta)$$

where we have assumed that $\delta\theta \ll \theta$. Multiplying the expressions,

$$n\sin\theta_2 = n\sin\theta_2 + n\cos\theta_2 \delta\theta + \delta n\sin\theta_2 + \delta n\delta\theta\cos\theta_2$$

We can ignore the last term on the right-hand side because it is the product of two small terms. The equation becomes

$$n\cos\theta_2 \delta\theta = -\delta n\sin\theta_2 \Rightarrow \delta\theta = -\tan\theta_2\left(\frac{\delta n}{n}\right)$$

Note that $\delta\theta$ has to be in radians.
We can obtain the same result in the following way as well. From Snell's law,

$$\sin\theta_2 = \frac{\sin\theta_1}{n}$$

Differentiating relative to n

$$\delta\left(\frac{\sin\theta_2}{\delta n}\right) = \cos\theta_2 \frac{\delta\theta}{\delta n} = (\sin\theta_1)\left(-\frac{1}{n^2}\right) = -\frac{n\sin\theta_2}{n^2} = \frac{-\sin\theta_2}{n}$$

$$\Rightarrow \delta\theta = -\tan\theta_2\left(\frac{\delta n}{n}\right)$$

(b) We have $\theta_1 = 30°$ and $n_{red} = 1.552$. Because the red wavelength is larger than the violet wavelength, $n_{red} < n_{violet}$. Also, if the refraction angle for the red light is θ_2, the refraction angle for the violet is less than θ_2. Thus, $\delta\theta = -0.28°$. From the formula obtained in part (a),

$$\delta\theta = -\tan\theta_2\left(\frac{\delta n}{n}\right) \Rightarrow \delta n = -\frac{n}{\tan\theta_2}\delta\theta$$

To determine $\tan\theta_2$, we note that

$$n_{red}\sin\theta_2 = n_{air}\sin 30° \Rightarrow \theta_2 = \sin^{-1}\left(\frac{\sin 30°}{1.552}\right) = 18.794° \Rightarrow \tan\theta_2 = 0.3403$$

Thus, the expression for the change in the index of refraction is

$$\delta n = -\frac{1.552}{0.3403}(-0.28°)\left(\frac{\pi \text{ rad}}{180°}\right) = 0.0223 \Rightarrow n_{violet} = n_{red} + \delta n = 1.552 + 0.022 = 1.574$$

23.83. **Visualize:** The lateral magnification is $m = -s'/s$.

Solve: We need to solve the thin-lens equation for s'.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$$

$$s' = \frac{1}{\dfrac{1}{f} - \dfrac{1}{s}} = \frac{fs}{s - f}$$

Now insert this expression for s' into the expression for m.

$$m = -\frac{s'}{s} = -\frac{\dfrac{fs}{s-f}}{s} = -\frac{f}{s-f}$$

Now define the longitudinal magnification as the rate of change of s' with respect to s.

$$M = \frac{ds'}{ds} = \frac{d}{ds}(s') = \frac{d}{ds}\left(\frac{fs}{s-f}\right)$$

Use the quotient rule of differentiation.

$$M = \frac{d}{ds}\left(\frac{fs}{s-f}\right) = \frac{(s-f)f - fs}{(s-f)^2} = \frac{-f^2}{(s-f)^2}$$

This last result is equal to $-m^2$, so $M = -m^2$.

Assess: For further reading, see "Longitudinal Magnification Drawing Mistake" by Héctor Rabal, Nelly Cam, and Marcelo Trivi in *The Physics Teacher*, vol. 42, January 2004, pp. 31–33, but know that Equation 4 there is missing a couple of minus signs.

23.84. **Model:** Assume the lens is thin so we can solve the thin-lens equation for $s' = \dfrac{fs}{s-f}$.

Visualize: We are given $f = 0.150$ m and $s = 10$ m. We are also given $ds/dt = 5.0$ m/s. Refer to Problem 23.83 to learn about longitudinal magnification: $M = ds'/ds = -m^2$.

Solve: The speed of the image is ds'/dt. Use the chain rule. Then apply $m = -s'/s$.

$$\frac{ds'}{dt} = \frac{ds'}{ds}\frac{ds}{dt} = M\frac{ds}{dt} = -m^2\frac{ds}{dt} = -\left(\frac{s'}{s}\right)^2\frac{ds}{dt} = -\left(\frac{\frac{fs}{s-f}}{s}\right)^2\frac{ds}{dt} = -\left(\frac{f}{s-f}\right)^2\frac{ds}{dt}$$

$$\left.\frac{ds'}{dt}\right|_{s=10\text{ m}} = -\left(\frac{f}{s-f}\right)^2\left.\frac{ds}{dt}\right|_{s=10\text{ m}} = \left(\frac{0.15\text{ m}}{10\text{ m} - 0.15\text{ m}}\right)^2(5.0\text{ m/s}) = -1.16\text{ mm/s}$$

Assess: The negative answer indicates the image is moving toward the lens.