14.1. Solve: The frequency generated by a guitar string is 440 Hz. The period is the inverse of the frequency, hence

$$T = \frac{1}{f} = \frac{1}{440\ \text{Hz}} = 2.27 \times 10^{-3}\ \text{s} = 2.27\ \text{ms}$$

14.2. Model: The air-track glider oscillating on a spring is in simple harmonic motion.

Solve: The glider completes 10 oscillations in 33 s, and it oscillates between the 10 cm mark and the 60 cm mark.

(a)
$$T = \frac{33\ \text{s}}{10\ \text{oscillations}} = 3.3\ \text{s/oscillation} = 3.3\ \text{s}$$

(b)
$$f = \frac{1}{T} = \frac{1}{3.3\ \text{s}} = 0.303\ \text{Hz} \approx 0.30\ \text{Hz}$$

(c)
$$\omega = 2\pi f = 2\pi(0.303\ \text{Hz}) = 1.90\ \text{rad/s}$$

(d) The oscillation from one side to the other is equal to $60\ \text{cm} - 10\ \text{cm} = 50\ \text{cm} = 0.50\ \text{m}$. Thus, the amplitude is $A = \frac{1}{2}(0.50\ \text{m}) = 0.25\ \text{m}$.

(e) The maximum speed is

$$v_{\max} = \omega A = \left(\frac{2\pi}{T}\right)A = (1.90\ \text{rad/s})(0.25\ \text{m}) = 0.48\ \text{m/s}$$

14.3. Model: The air-track glider attached to a spring is in simple harmonic motion.

Visualize: The position of the glider can be represented as $x(t) = A\cos\omega t$.

Solve: The glider is pulled to the right and released from rest at $t = 0$ s. It then oscillates with a period $T = 2.0\ \text{s}$ and a maximum speed $v_{\text{max}} = 40\ \text{cm/s} = 0.40\ \text{m/s}$.

(a) $v_{\text{max}} = \omega A$ and $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2.0\ \text{s}} = \pi\ \text{rad/s} \Rightarrow A = \dfrac{v_{\text{max}}}{\omega} = \dfrac{0.40\ \text{m/s}}{\pi\ \text{rad/s}} = 0.127\ \text{m} = 12.7\ \text{cm}$

(b) The glider's position at $t = 0.25\ \text{s}$ is

$$x_{0.25\ \text{s}} = (0.127\ \text{m})\cos\left[(\pi\ \text{rad/s})(0.25\ \text{s})\right] = 0.090\ \text{m} = 9.0\ \text{cm}$$

14.4. Model: The oscillation is the result of simple harmonic motion.
Visualize: Please refer to Figure EX14.4.
Solve: **(a)** The amplitude $A = 10$ cm.
(b) The time to complete one cycle is the period, hence $T = 2.0$ s and

$$f = \frac{1}{T} = \frac{1}{2.0\text{ s}} = 0.50\text{ Hz}$$

(c) The position of an object undergoing simple harmonic motion is $x(t) = A\cos(\omega t + \phi_0)$. At $t = 0$ s, $x_0 = -5$ cm, thus

$$-5\text{ cm} = (10\text{ cm})\cos\left[\omega(0\text{ s}) + \phi_0\right]$$
$$\Rightarrow \cos\phi_0 = \frac{-5\text{ cm}}{10\text{ cm}} = -\frac{1}{2} \Rightarrow \phi_0 = \cos^{-1}\left(-\frac{1}{2}\right) = \pm\frac{2\pi}{3}\text{ rad or } \pm 120°$$

Since the oscillation is originally moving to the left, $\phi_0 = +120°$.

14.5. Model: The oscillation is the result of simple harmonic motion.
Visualize: Please refer to Figure EX14.5.
Solve: (a) The amplitude $A = 20$ cm.
(b) The period $T = 4.0$ s, thus

$$f = \frac{1}{T} = \frac{1}{4.0\text{ s}} = 0.25\text{ Hz}$$

(c) The position of an object undergoing simple harmonic motion is $x(t) = A\cos(\omega t + \phi_0)$. At $t = 0$ s, $x_0 = 10$ cm. Thus,

$$10\text{ cm} = (20\text{ cm})\cos\phi_0 \Rightarrow \phi_0 = \cos^{-1}\left(\frac{10\text{ cm}}{20\text{ cm}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \pm\frac{\pi}{3}\text{ rad} = \pm 60°$$

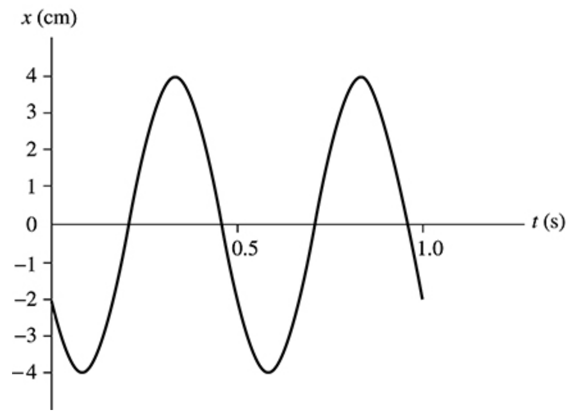
Because the object is moving to the right at $t = 0$ s, it is in the lower half of the circular motion diagram and thus must have a phase constant between π and 2π radians. Therefore, $\phi_0 = -\frac{\pi}{3}$ rad $= -60°$.

14.6. Visualize: The phase constant $\frac{2}{3}\pi$ has a plus sign, which implies that the object undergoing simple harmonic motion is in the second quadrant of the circular motion diagram. That is, the object is moving to the left.

Solve: The position of the object is

$$x(t) = A\cos(\omega t + \phi_0) = A\cos(2\pi ft + \phi_0) = (4.0\text{ cm})\cos\left[(4\pi \text{ rad/s})t + \tfrac{2}{3}\pi \text{ rad}\right]$$

The amplitude is $A = 4$ cm and the period is $T = 1/f = 0.50$ s. A phase constant $\phi_0 = 2\pi/3$ rad $=120°$ (second quadrant) means that x starts at $-\frac{1}{2}A$ and is moving to the left (getting more negative).



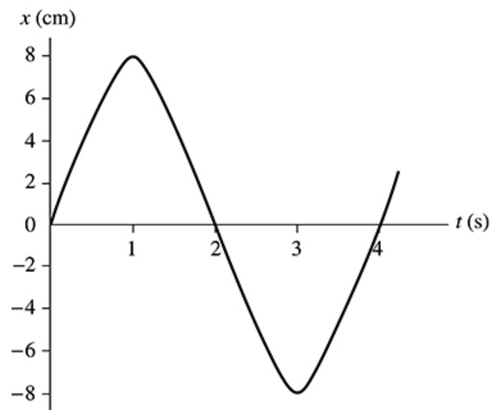
Assess: We can see from the graph that the object starts out moving to the left.

14.7. Visualize: A phase constant of $-\dfrac{\pi}{2}$ implies that the object that undergoes simple harmonic motion is in the lower half of the circular motion diagram. That is, the object is moving to the right.

Solve: The position of the object is given by the equation

$$x(t) = A\cos(\omega t + \phi_0) = A\cos(2\pi ft + \phi_0) = (8.0\text{ cm})\cos\left[\left(\frac{\pi}{2}\text{ rad/s}\right)t - \frac{\pi}{2}\text{ rad}\right]$$

The amplitude is $A = 8.0$ cm and the period is $T = 1/f = 4.0$ s. With $\phi_0 = -\pi/2$ rad, x starts at 0 cm and is moving to the right (getting more positive).



Assess: As we see from the graph, the object starts out moving to the right.

14.8. Solve: The position of the object is given by the equation

$$x(t) = A\cos(\omega t + \phi_0) = A\cos(2\pi ft + \phi_0)$$

We can find the phase constant ϕ_0 from the initial condition:

$$0 \text{ cm} = (4.0 \text{ cm})\cos\phi_0 \Rightarrow \cos\phi_0 = 0 \Rightarrow \phi_0 = \cos^{-1}(0) = \pm\tfrac{1}{2}\pi \text{ rad}$$

Since the object is moving to the right, the object is in the lower half of the circular motion diagram. Hence, $\phi_0 = -\frac{1}{2}\pi$ rad. The final result, with $f = 4.0$ Hz, is

$$x(t) = (4.0 \text{ cm})\cos\left[(8.0\pi \text{ rad/s})t - \tfrac{1}{2}\pi \text{ rad}\right]$$

14.9. Solve: The position of the object is given by the equation

$$x(t) = A\cos(\omega t + \phi_0)$$

The amplitude $A = 8.0$ cm. The angular frequency $\omega = 2\pi f = 2\pi(0.50 \text{ Hz}) = \pi$ rad/s. Since at $t = 0$ it has its most negative velocity, it must be at the equilibrium point $x = 0$ cm and moving to the left, so $\phi_0 = \frac{\pi}{2}$. Thus

$$x(t) = (8.0 \text{ cm})\cos[(\pi \text{ rad/s})t + \frac{\pi}{2} \text{ rad}]$$

14.10. Model: The air-track glider is in simple harmonic motion.

Solve: **(a)** We can find the phase constant from the initial conditions for position and velocity:

$$x_0 = A\cos\phi_0 \quad v_{0x} = -\omega A\sin\phi_0$$

Dividing the second by the first, we see that

$$\frac{\sin\phi_0}{\cos\phi_0} = \tan\phi_0 = -\frac{v_{0x}}{\omega x_0}$$

The glider starts to the left ($x_0 = -5.00$ cm) and is moving to the right ($v_{0x} = +36.3$ cm/s). With a period of $1.5\text{ s} = \frac{3}{2}\text{ s}$, the angular frequency is $\omega = 2\pi/T = \frac{4}{3}\pi$ rad/s. Thus

$$\phi_0 = \tan^{-1}\left(-\frac{36.3\text{ cm/s}}{(4\pi/3\text{ rad/s})(-5.00\text{ cm})}\right) = \tfrac{1}{3}\pi\text{ rad }(60°)\text{ or } -\tfrac{2}{3}\pi\text{ rad }(-120°)$$

The tangent function repeats every 180°, so there are always two possible values when evaluating the arctan function. We can distinguish between them because an object with a negative position but moving to the right is in the third quadrant of the corresponding circular motion. Thus $\phi_0 = -\frac{2}{3}\pi$ rad, or $-120°$.

(b) At time t, the phase is $\phi = \omega t + \phi_0 = (\frac{4}{3}\pi\text{ rad/s})t - \frac{2}{3}\pi$ rad. This gives $\phi = -\frac{2}{3}\pi$ rad, 0 rad, $\frac{2}{3}\pi$ rad, and $\frac{4}{3}\pi$ rad at, respectively, $t = 0$ s, 0.5 s, 1.0 s, and 1.5 s. This is one period of the motion.

14.11. Model: The block attached to the spring is in simple harmonic motion.
Solve: The period of an object attached to a spring is

$$T = 2\pi\sqrt{\frac{m}{k}} = T_0 = 2.0 \text{ s}$$

where m is the mass and k is the spring constant.
(a) For mass $= 2m$,

$$T = 2\pi\sqrt{\frac{2m}{k}} = \left(\sqrt{2}\right)T_0 = 2.8 \text{ s}$$

(b) For mass $\frac{1}{2}m$,

$$T = 2\pi\sqrt{\frac{\frac{1}{2}m}{k}} = T_0 \big/ \sqrt{2} = 1.41 \text{ s}$$

(c) The period is independent of amplitude. Thus $T = T_0 = 2.0$ s
(d) For a spring constant $= 2k$,

$$T = 2\pi\sqrt{\frac{m}{2k}} = T_0 \big/ \sqrt{2} = 1.41 \text{ s}$$

14.12. Model: The air-track glider attached to a spring is in simple harmonic motion.

Solve: Experimentally, the period is $T = (12.0 \text{ s})/(10 \text{ oscillations}) = 1.20 \text{ s}$. Using the formula for the period,

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \left(\frac{2\pi}{T}\right)^2 m = \left(\frac{2\pi}{1.20 \text{ s}}\right)^2 (0.200 \text{ kg}) = 5.48 \text{ N/m}$$

14.13. Model: The mass attached to the spring oscillates in simple harmonic motion.

Solve: **(a)** The period $T = 1/f = 1/2.0 \text{ Hz} = 0.50 \text{ s}$.

(b) The angular frequency $\omega = 2\pi f = 2\pi(2.0 \text{ Hz}) = 4\pi \text{ rad/s}$.

(c) Using energy conservation

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}kx_0^2 + \tfrac{1}{2}mv_{0x}^2$$

Using $x_0 = 5.0 \text{ cm}$, $v_{0x} = -30 \text{ cm/s}$ and $k = m\omega^2 = (0.200 \text{ kg})(4\pi \text{ rad/s})^2$, we get $A = 5.54 \text{ cm}$.

(d) To calculate the phase constant ϕ_0,

$$A\cos\phi_0 = x_0 = 5.0 \text{ cm}$$

$$\Rightarrow \phi_0 = \cos^{-1}\left(\frac{5.0 \text{ cm}}{5.54 \text{ cm}}\right) = 0.45 \text{ rad}$$

(e) The maximum speed is $v_{\max} = \omega A = (4\pi \text{ rad/s})(5.54 \text{ cm}) = 70 \text{ cm/s}$.

(f) The maximum acceleration is

$$a_{\max} = \omega^2 A = \omega(\omega A) = (4\pi \text{ rad/s})(70 \text{ cm/s}) = 8.8 \text{ m/s}^2$$

(g) The total energy is $E = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}(0.200 \text{ kg})(0.70 \text{ m/s})^2 = 0.049 \text{ J}$.

(h) The position at $t = 0.40 \text{ s}$ is

$$x_{0.4 \text{ s}} = (5.54 \text{ cm})\cos\left[(4\pi \text{ rad/s})(0.40 \text{ s}) + 0.45 \text{ rad}\right] = +3.8 \text{ cm}$$

14.14. Model: The oscillating mass is in simple harmonic motion.

Solve: **(a)** The amplitude $A = 2.0$ cm.

(b) The period is calculated as follows:

$$\omega = \frac{2\pi}{T} = 10 \text{ rad/s} \Rightarrow T = \frac{2\pi}{10 \text{ rad/s}} = 0.63 \text{ s}$$

(c) The spring constant is calculated as follows:

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (0.050 \text{ kg})(10 \text{ rad/s})^2 = 5.0 \text{ N/m}$$

(d) The phase constant $\phi_0 = -\frac{1}{4}\pi$ rad.

(e) The initial conditions are obtained from the equations

$$x(t) = (2.0 \text{ cm})\cos\left(10t - \tfrac{1}{4}\pi\right) \text{ and } v_x(t) = -(20.0 \text{ cm/s})\sin\left(10t - \tfrac{1}{4}\pi\right)$$

At $t = 0$ s, these equations become

$$x_0 = (2.0 \text{ cm})\cos\left(-\tfrac{1}{4}\pi\right) = 1.41 \text{ cm and } v_{0x} = -(20 \text{ cm/s})\sin\left(-\tfrac{1}{4}\pi\right) = 14.1 \text{ cm/s}$$

In other words, the mass is at $+1.41$ cm and moving to the right with a velocity of 14.1 cm/s.

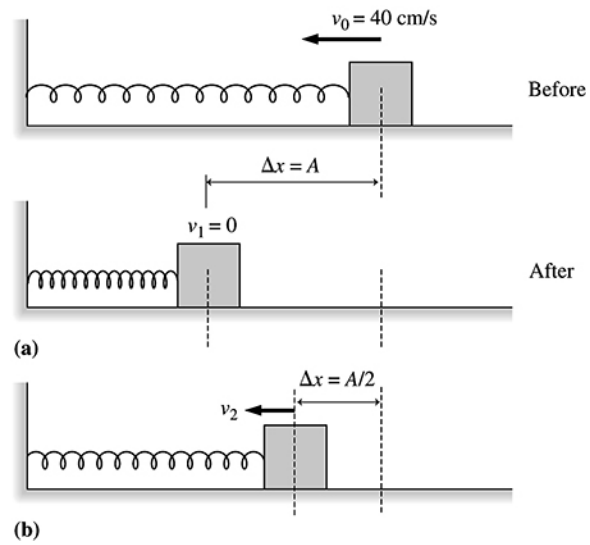
(f) The maximum speed is $v_{\max} = A\omega = (2.0 \text{ cm})(10 \text{ rad/s}) = 20 \text{ cm/s}$.

(g) The total energy $E = \frac{1}{2}kA^2 = \frac{1}{2}(5.0 \text{ N/m})(0.020 \text{ m})^2 = 1.00 \times 10^{-3}$ J.

(h) At $t = 0.41$ s, the velocity is

$$v_{0x} = -(20 \text{ cm/s})\sin\left[(10 \text{ rad/s})(0.40 \text{ s}) - \tfrac{1}{4}\pi\right] = 1.46 \text{ cm/s}$$

14.15. Model: The block attached to the spring is in simple harmonic motion.
Visualize:



Solve: **(a)** The conservation of mechanical energy equation $K_f + U_{sf} = K_i + U_{si}$ is

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}k(\Delta x)^2 = \tfrac{1}{2}mv_0^2 + 0\text{ J} \Rightarrow 0\text{ J} + \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_0^2 + 0\text{ J}$$

$$\Rightarrow A = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{1.0\text{ kg}}{16\text{ N/m}}}(0.40\text{ m/s}) = 0.10\text{ m} = 10.0\text{ cm}$$

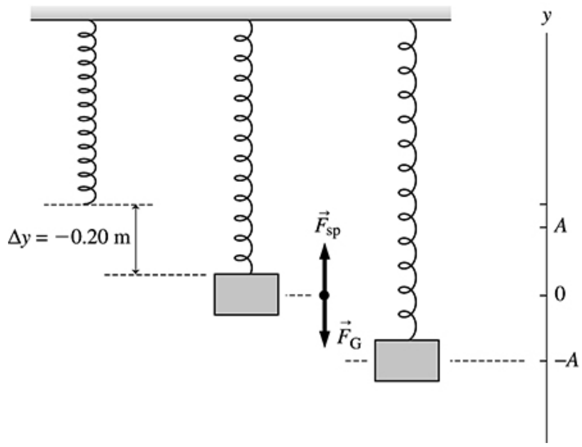
(b) We have to find the velocity at a point where $x = A/2$. The conservation of mechanical energy equation $K_2 + U_{s2} = K_i + U_{si}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}mv_0^2 + 0\text{ J} \Rightarrow \frac{1}{2}mv_2^2 = \frac{1}{2}mv_0^2 - \frac{1}{4}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}mv_0^2 - \frac{1}{4}\left(\frac{1}{2}mv_0^2\right) = \frac{3}{4}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow v_2 = \sqrt{\frac{3}{4}}v_0 = \sqrt{\frac{3}{4}}(0.40\text{ m/s}) = 0.346\text{ m/s}$$

The velocity is 35 cm/s.

14.16. Model: The vertical oscillations constitute simple harmonic motion.
Visualize:



Solve: (a) At equilibrium, Newton's first law applied to the physics book is

$$(F_{sp})_y - mg = 0 \text{ N} \Rightarrow -k\Delta y - mg = 0 \text{ N}$$
$$\Rightarrow k = -mg/\Delta y = -(0.500 \text{ kg})(9.8 \text{ m/s}^2)/(-0.20 \text{ m}) = 24.5 \text{ N/m}$$

(b) To calculate the period:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{24.5 \text{ N/m}}{0.500 \text{ kg}}} = 7.0 \text{ rad/s and } T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{7.0 \text{ rad/s}} = 0.90 \text{ s}$$

(c) The maximum speed is

$$v_{max} = A\omega = (0.10 \text{ m})(7.0 \text{ rad/s}) = 0.70 \text{ m/s}$$

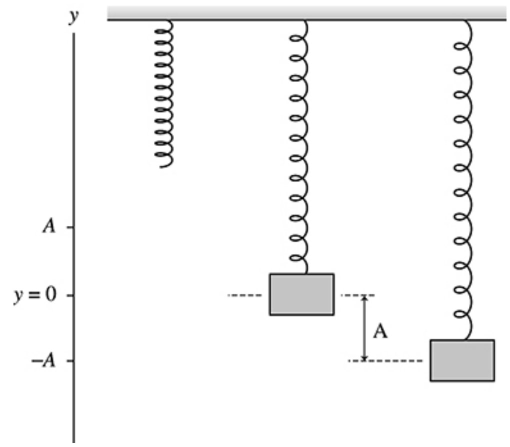
Maximum speed occurs as the book passes through the equilibrium position.

14.17. Model: The vertical oscillations constitute simple harmonic motion.

Solve: To find the oscillation frequency using $\omega = 2\pi f = \sqrt{k/m}$, we first need to find the spring constant *k*. In equilibrium, the weight mg of the block and the spring force $k\Delta L$ are equal and opposite. That is, $mg = k\Delta L \Rightarrow k = mg/\Delta L$. The frequency of oscillation *f* is thus given as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{mg/\Delta L}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\ \text{m/s}^2}{0.020\ \text{m}}} = 3.5\ \text{Hz}$$

14.18. Model: The vertical oscillations constitute simple harmonic motion.
Visualize:



Solve: The period and angular frequency are

$$T = \frac{20\text{ s}}{30\text{ oscillations}} = 0.6667\text{ s and } \omega = \frac{2\pi}{T} = \frac{2\pi}{0.6667\text{ s}} = 9.425\text{ rad/s}$$

(a) The mass can be found as follows:

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{k}{\omega^2} = \frac{15\text{ N/m}}{(9.425\text{ rad/s})^2} = 0.169\text{ kg}$$

(b) The maximum speed $v_{\text{max}} = \omega A = (9.425\text{ rad/s})(0.060\text{ m}) = 0.57\text{ m/s}$.

14.19. Model: Assume a small angle of oscillation so there is simple harmonic motion.
Solve: The period of the pendulum is

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}} = 4.0 \text{ s}$$

(a) The period is independent of the mass and depends only on the length. Thus $T = T_0 = 4.0$ s.
(b) For a new length $L = 2L_0$,

$$T = 2\pi\sqrt{\frac{2L_0}{g}} = \sqrt{2}T_0 = 5.7 \text{ s}$$

(c) For a new length $L = L_0/2$,

$$T = 2\pi\sqrt{\frac{L_0/2}{g}} = \frac{1}{\sqrt{2}}T_0 = 2.8 \text{ s}$$

(d) The period is independent of the amplitude as long as there is simple harmonic motion. Thus $T = 4.0$ s.

14.20. Model: The pendulum undergoes simple harmonic motion.
Solve: **(a)** The amplitude is 0.10 rad.
(b) The frequency of oscillations is

$$f = \frac{\omega}{2\pi} = \frac{5}{2\pi}\ \text{Hz} = 0.796\ \text{Hz}$$

(c) The phase constant $\phi = \pi$ rad.
(d) The length can be obtained from the period:

$$\omega = 2\pi f = \sqrt{\frac{g}{L}} \Rightarrow L = \left(\frac{1}{2\pi f}\right)^2 g = \left(\frac{1}{2\pi(0.796\ \text{Hz})}\right)^2 (9.8\ \text{m/s}^2) = 0.392\ \text{m}$$

(e) At $t = 0$ s, $\theta_0 = (0.10\ \text{rad})\cos(\pi) = -0.10$ rad. To find the initial condition for the angular velocity we take the derivative of the angular position:

$$\theta(t) = (0.10\ \text{rad})\cos(5t + \pi) \Rightarrow \frac{d\theta(t)}{dt} = -(0.10\ \text{rad})(5)\sin(5t + \pi)$$

At $t = 0$ s, $(d\theta/dt)_0 = (-0.50\ \text{rad})\sin(\pi) = 0\ \text{rad/s}$.
(f) At $t = 2.0$ s, $\theta_{2.0} = (0.10\ \text{rad})\cos(5(2.0\ \text{s}) + \pi) = 0.084$ rad.

14.21. Model: Assume the small-angle approximation so there is simple harmonic motion.
Solve: The period is $T = 12\ \text{s}/10\ \text{oscillations} = 1.20\ \text{s}$ and is given by the formula

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{1.20\ \text{s}}{2\pi}\right)^2 \left(9.8\ \text{m/s}^2\right) = 36\ \text{cm}$$

14.22. Model: Assume a small angle of oscillation so there is simple harmonic motion.
Solve: **(a)** On the earth the period is

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.0 \text{ m}}{9.80 \text{ m/s}^2}} = 2.0 \text{ s}$$

(b) On Venus the acceleration due to gravity is

$$g_{\text{Venus}} = \frac{GM_{\text{Venus}}}{R_{\text{Venus}}^2} = \frac{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.88\times10^{24} \text{ kg}\right)}{\left(6.06\times10^{6} \text{ m}\right)^2} = 8.86 \text{ m/s}^2$$

$$\Rightarrow T_{\text{Venus}} = 2\pi\sqrt{\frac{L}{g_{\text{Venus}}}} = 2\pi\sqrt{\frac{1.0 \text{ m}}{8.86 \text{ m/s}^2}} = 2.1 \text{ s}$$

14.23. Model: Assume the pendulum to have small-angle oscillations. In this case, the pendulum undergoes simple harmonic motion.

Solve: Using the formula $g = GM/R^2$, the periods of the pendulums on the moon and on the earth are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L_{\text{earth}}R_{\text{earth}}^2}{GM_{\text{earth}}}} \text{ and } T_{\text{moon}} = 2\pi\sqrt{\frac{L_{\text{moon}}R_{\text{moon}}^2}{GM_{\text{moon}}}}$$

Because $T_{\text{earth}} = T_{\text{moon}}$,

$$2\pi\sqrt{\frac{L_{\text{earth}}R_{\text{earth}}^2}{GM_{\text{earth}}}} = 2\pi\sqrt{\frac{L_{\text{moon}}R_{\text{moon}}^2}{GM_{\text{moon}}}} \Rightarrow L_{\text{moon}} = \left(\frac{M_{\text{moon}}}{M_{\text{earth}}}\right)\left(\frac{R_{\text{earth}}}{R_{\text{moon}}}\right)^2 L_{\text{earth}}$$

$$= \left(\frac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{6.37\times10^{6}\ \text{m}}{1.74\times10^{6}\ \text{m}}\right)^2 (2.0\ \text{m}) = 33\ \text{cm}$$

14.24. Model: Assume a small angle of oscillation so that the pendulum has simple harmonic motion.
Solve: The time periods of the pendulums on the earth and on Mars are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g_{\text{earth}}}} \text{ and } T_{\text{Mars}} = 2\pi\sqrt{\frac{L}{g_{\text{Mars}}}}$$

Dividing these two equations,

$$\frac{T_{\text{earth}}}{T_{\text{Mars}}} = \sqrt{\frac{g_{\text{Mars}}}{g_{\text{earth}}}} \Rightarrow g_{\text{Mars}} = g_{\text{earth}}\left(\frac{T_{\text{earth}}}{T_{\text{Mars}}}\right)^2 = \left(9.8 \text{ m/s}^2\right)\left(\frac{1.50 \text{ s}}{2.45 \text{ s}}\right)^2 = 3.67 \text{ m/s}^2$$

14.25. Visualize: Please refer to Figure Ex14.25.
Solve: The mass of the wrench can be obtained from the length that it stretches the spring. From Equation 14.41,

$$\Delta L = \frac{mg}{k} \Rightarrow m = \frac{k\Delta L}{g} = \frac{360\ \text{N/m}\left(0.030\ \text{m}\right)}{9.8\ \text{m/s}^2} = 1.10\ \text{kg}$$

When swinging on a hook the wrench is a physical pendulum. From Equation 14.52,

$$2\pi f = \sqrt{\frac{mgl}{I}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{mgl}{I}}$$

From the figure, $l = 0.14$ m. Thus

$$I = \left(\frac{T}{2\pi}\right)^2 mgl = \left(\frac{0.90\ \text{s}}{2\pi}\right)^2 \left(1.10\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\left(0.14\ \text{m}\right) = 3.1\times10^{-2}\ \text{kg m}^2$$

14.26. **Model:** The spider is in simple harmonic motion.

Solve: Your tapping is a driving frequency. Largest amplitude at $f_{\text{ext}} = 1.0$ Hz means that this is the resonance frequency, so $f_0 = f_{\text{ext}} = 1.0$ Hz. That is, the spider's natural frequency of oscillation f_0 is 1.0 Hz and $\omega_0 = 2\pi f_0 = 2\pi$ rad/s. We have

$$\omega_0 = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega_0^2 = (0.0020\ \text{kg})(2\pi\ \text{rad/s})^2 = 0.079\ \text{N/m}$$

14.27. **Model:** The motion is a damped oscillation.

Solve: The amplitude of the oscillation at time t is given by Equation 14.58: $A(t) = A_0 e^{-t/2\tau}$, where $\tau = m/b$ is the time constant. Using $x = 0.368$ A and $t = 10.0$ s, we get

$$0.368A = Ae^{-10.0\ \text{s}/2\tau} \Rightarrow \ln(0.368) = \frac{-10\ \text{s}}{2\tau} \Rightarrow \tau = -\frac{10.0\ \text{s}}{2\ln(0.368)} = 5.00\ \text{s}$$

14.28. Model: The motion is a damped oscillation.

Solve: The position of a damped oscillator is $x(t) = Ae^{-(t/2\tau)}\cos(\omega t + \phi_0)$. The frequency is 1.0 Hz and the damping time constant τ is 4.0 s. Let us assume $\phi_0 = 0$ rad and $A = 1$ with arbitrary units. Thus,

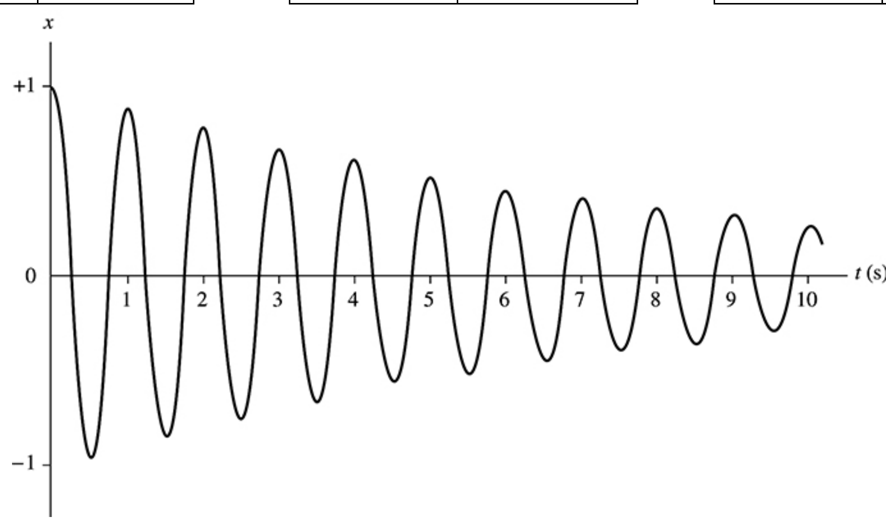
$$x(t) = e^{-t/(8.0\ \mathrm{s})}\cos\left[2\pi(1.0\ \mathrm{Hz})t\right] \Rightarrow x(t) = e^{-0.125\ \mathrm{t}}\cos(2\pi t)$$

where *t* is in s. Values of *x*(*t*) at selected values of *t* are displayed in the following table:

t(s)	*x*(*t*)
0	1
0.25	0
0.50	−0.939
0.75	0
1.00	0.882
1.25	0
1.50	−0.829
1.75	0

t(s)	*x*(*t*)
2.00	0.779
2.50	−0.732
3.00	0.687
3.50	−0.646
4.00	0.607
4.50	−0.570
5.00	0.535
5.50	−0.503

t(s)	*x*(*t*)
6.00	0.472
6.50	− 0.444
7.00	0.417
7.50	− 0.392
8.00	0.368
8.50	− 0.346
9.00	0.325
9.50	− 0.305
10.00	0.286



14.29. Model: The pendulum is a damped oscillator.

Solve: The period of the pendulum and the number of oscillations in 4 hours are calculated as follows:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{15.0 \text{ m}}{9.8 \text{ m/s}^2}} = 7.773 \text{ s} \Rightarrow N_{\text{osc}} = \frac{4(3600 \text{ s})}{7.773 \text{ s}} = 1853$$

The amplitude of the pendulum as a function of time is $A(t) = Ae^{-bt/2m}$. The exponent of this expression can be calculated to be

$$-\frac{bt}{2m} = -\frac{(0.010 \text{ kg/s})(4\times 3600 \text{ s})}{2(110 \text{ kg})} = -0.6545$$

We have $A(t) = (1.50 \text{ m})e^{-0.6545} = 0.780 \text{ m}$.

14.30. Model: The vertical oscillations are damped and follow simple harmonic motion.

Solve: The position of the ball is given by $x(t) = Ae^{-(t/2\tau)}\cos(\omega t + \phi_0)$. The amplitude $A(t) = Ae^{-(t/2\tau)}$ is a function of time. The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{(15.0\ \text{N/m})}{0.500\ \text{kg}}} = 5.477\ \text{rad/s} \Rightarrow T = \frac{2\pi}{\omega} = 1.147\ \text{s}$$

Because the ball's amplitude decreases to 3.0 cm from 6.0 cm after 30 oscillations, that is, after $30 \times 1.147\ \text{s} =$ 34.41 s, we have

$$3.0\ \text{cm} = (6.0\ \text{cm})e^{-(34.414\ \text{s}/2\tau)} \Rightarrow 0.50 = e^{-(34.41\ \text{s}/2\tau)} \Rightarrow \ln(0.50) = \frac{-34.41\ \text{s}}{2\tau} \Rightarrow \tau = 25\ \text{s}$$

14.31. Visualize: Please refer to Figure P14.31.

Solve: The position and the velocity of a particle in simple harmonic motion are

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0) = -v_{\max}\sin(\omega t + \phi_0)$$

(a) At $t = 0$ s, the equation for x yields

$$(-5.0 \text{ cm}) = (10.0 \text{ cm})\cos(\phi_0) \Rightarrow \phi_0 = \cos^{-1}(-0.5) = \pm\tfrac{2}{3}\pi \text{ rad}$$

Because the particle is moving to the left at $t = 0$ s, it is in the upper half of the circular motion diagram, and the phase constant is between 0 and π radians. Thus, $\phi_0 = \tfrac{2}{3}\pi$ rad.

(b) The period is 4.0 s. At $t = 0$ s,

$$v_{0x} = -A\omega\sin\phi_0 = -(10.0 \text{ cm})\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi}{3}\right) = -13.6 \text{ cm/s}$$

(c) The maximum speed is

$$v_{\max} = \omega A = \left(\frac{2\pi}{4.0 \text{ s}}\right)(10.0 \text{ cm}) = 15.7 \text{ cm/s}$$

Assess: The negative velocity at $t = 0$ s is consistent with the position-versus-time graph and the positive sign of the phase constant.

14.32. Visualize: Please refer to Figure P14.32.
Solve: The position and the velocity of a particle in simple harmonic motion are

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0) = -v_{\max}\sin(\omega t + \phi_0)$$

From the graph, $T = 12$ s and the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{12 \text{ s}} = \frac{\pi}{6} \text{ rad/s}$$

(a) Because $v_{\max} = A\omega = 60$ cm/s, we have

$$A = \frac{60 \text{ cm/s}}{\omega} = \frac{60 \text{ cm/s}}{\pi/6 \text{ rad/s}} = 115 \text{ cm}$$

(b) At $t = 0$ s,

$$v_{0x} = -A\omega\sin\phi_0 = -30 \text{ cm/s} \Rightarrow -(60 \text{ cm/s})\sin\phi_0 = -30 \text{ cm/s}$$
$$\Rightarrow \phi_0 = \sin^{-1}(0.5 \text{ rad}) = \tfrac{1}{6}\pi \text{ rad } (30°) \text{ or } \tfrac{5}{6}\pi \text{ rad } (150°)$$

Because the velocity at $t = 0$ s is negative and the particle is slowing down, the particle is in the second quadrant of the circular motion diagram. Thus $\phi_0 = \frac{5}{6}\pi$ rad.
(c) At $t = 0$ s, $x_0 = (115 \text{ cm})\cos\left(\frac{5}{6}\pi \text{ rad}\right) = -100$ cm.

14.33. Model: The vertical mass/spring systems are in simple harmonic motion.
Visualize: Please refer to Figure P14.33.
Solve: (a) For system A, the maximum speed while traveling in the upward direction corresponds to the maximum positive slope, which is at $t = 3.0$ s. The frequency of oscillation is 0.25 Hz.
(b) For system B, all the energy is potential energy when the position is at maximum amplitude, which for the first time is at $t = 1.5$ s. The time period of system B is thus 6.0 s.
(c) Spring/mass A undergoes three oscillations in 12 s, giving it a period $T_A = 4.0$ s. Spring/mass B undergoes 2 oscillations in 12 s, giving it a period $T_B = 6.0$ s. We have

$$T_A = 2\pi\sqrt{\frac{m_A}{k_A}} \text{ and } T_B = 2\pi\sqrt{\frac{m_B}{k_B}} \Rightarrow \frac{T_A}{T_B} = \sqrt{\left(\frac{m_A}{m_B}\right)\left(\frac{k_B}{k_A}\right)} = \frac{4.0 \text{ s}}{6.0 \text{ s}} = \frac{2}{3}$$

If $m_A = m_B$, then

$$\frac{k_B}{k_A} = \frac{4}{9} \Rightarrow \frac{k_A}{k_B} = \frac{9}{4} = 2.25$$

14.34. Solve: The object's position as a function of time is $x(t) = A\cos(\omega t + \phi_0)$. Letting $x = 0$ m at $t = 0$ s, gives

$$0 = A\cos\phi_0 \Rightarrow \phi_0 = \pm\tfrac{1}{2}\pi$$

Since the object is traveling to the right, it is in the lower half of the circular motion diagram, giving a phase constant between $-\pi$ and 0 radians. Thus, $\phi_0 = -\tfrac{1}{2}\pi$ and

$$x(t) = A\cos\left(\omega t - \tfrac{1}{2}\pi\right) \Rightarrow x(t) = A\sin\omega t = (0.10\text{ m})\sin\left(\tfrac{1}{2}\pi t\right)$$

where we have used $A = 0.10$ m and

$$\omega = \frac{2\pi}{T} = \frac{2\pi\text{ rad}}{4.0\text{ s}} = \frac{\pi}{2}\text{ rad/s}$$

Let us now find t where $x = 0.60$ m:

$$0.060\text{ m} = (0.10\text{ m})\sin\left(\frac{\pi}{2}t\right) \Rightarrow t = \frac{2}{\pi}\sin^{-1}\left(\frac{0.060\text{ m}}{0.10\text{ m}}\right) = 0.41\text{ s}$$

Assess: The answer is reasonable because it is approximately $\frac{1}{8}$ of the period.

14.35. Model: The block attached to the spring is in simple harmonic motion.
Visualize: The position and the velocity of the block are given by the equations

$$x(t) = A\cos(\omega t + \phi_0) \text{ and } v_x(t) = -A\omega\sin(\omega t + \phi_0)$$

Solve: To graph $x(t)$ we need to determine ω, ϕ_0, and A. These quantities will be found by using the initial $(t = 0 \text{ s})$ conditions on $x(t)$ and $v_x(t)$. The period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0 \text{ kg}}{20 \text{ N/m}}} = 1.405 \text{ s} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{1.405 \text{ s}} = 4.472 \text{ rad/s}$$

At $t = 0$ s, $x_0 = A\cos\phi_0$ and $v_{0x} = -A\omega\sin\phi_0$. Dividing these equations,

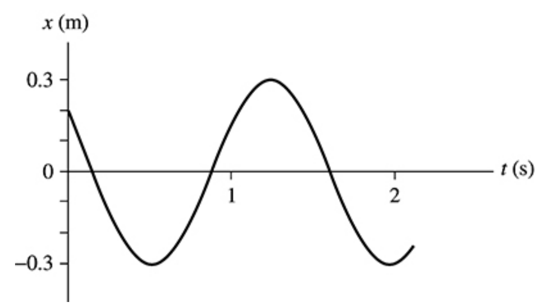
$$\tan\phi_0 = -\frac{v_{0x}}{\omega x_0} = -\frac{(-1.0 \text{ m/s})}{(4.472 \text{ rad/s})(0.20 \text{ m})} = 1.1181 \Rightarrow \phi_0 = 0.841 \text{ rad}$$

From the initial conditions,

$$A = \sqrt{x_0^2 + \left(\frac{v_{0x}}{\omega}\right)^2} = \sqrt{(0.20 \text{ m})^2 + \left(\frac{-1.0 \text{ m/s}}{4.472 \text{ rad/s}}\right)^2} = 0.300 \text{ m}$$

The position-versus-time graph can now be plotted using the equation

$$x(t) = (0.300 \text{ m})\cos[(4.472 \text{ rad/s})t + 0.841 \text{ rad}]$$



14.36. Model: The astronaut attached to the spring is in simple harmonic motion.
Visualize: Please refer to Figure P14.36.
Solve: **(a)** From the graph, $T = 3.0\text{ s}$, so we have

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \left(\frac{T}{2\pi}\right)^2 k = \left(\frac{3.0\text{ s}}{2\pi}\right)^2 (240\text{ N/m}) = 55\text{ kg}$$

(b) Oscillations occur about an equilibrium position of 1.0 m. From the graph, $A = \frac{1}{2}(0.80\text{ m}) = 0.40\text{ m}$, $\phi_0 =$ 0 rad, and

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.0\text{ s}} = 2.1\text{ rad/s}$$

The equation for the position of the astronaut is

$$x(t) = A\cos\omega t + 1.0\text{ m} = (0.4\text{ m})\cos\left[(2.1\text{ rad/s})t\right] + 1.0\text{ m}$$
$$\Rightarrow 1.2\text{ m} = (0.4\text{ m})\cos\left[(2.1\text{ rad/s})t\right] + 1.0\text{ m} \Rightarrow \cos\left[(2.1\text{ rad/s})t\right] = 0.5 \Rightarrow t = 0.50\text{ s}$$

The equation for the velocity of the astronaut is

$$v_x(t) = -A\omega\sin(\omega t)$$
$$\Rightarrow v_{0.5\text{ s}} = -(0.4\text{ m})(2.1\text{ rad/s})\sin\left[(2.1\text{ rad/s})(0.50\text{ s})\right] = -0.73\text{ m/s}$$

Thus her speed is 0.73 m/s.

14.37. Model: The particle is in simple harmonic motion.

Solve: The equation for the velocity of the particle is

$$v_x(t) = -(25\ \text{cm})(10\ \text{rad/s})\sin(10\ \text{t})$$

Substituting into $K = 2U$ gives

$$\frac{1}{2}mv_x^2(t) = 2\left(\frac{1}{2}kx^2(t)\right) \Rightarrow \frac{1}{2}m\left[-(250\ \text{cm/s})\sin(10\ \text{t})\right]^2 = k\left[(25\ \text{cm})\cos(10\ \text{t})\right]^2$$

$$\Rightarrow \frac{\sin^2(10\ \text{t})}{\cos^2(10\ \text{t})} = 2\left(\frac{k}{m}\right)\frac{(25\ \text{cm})^2}{(250\ \text{cm/s})^2} = 2\omega^2\left(\frac{1}{100}\right)\ \text{s}^2$$

$$\Rightarrow \tan^2(10\ \text{t}) = 2(10\ \text{rad/s})^2\left(\frac{1}{100}\right)\ \text{s}^2 = 2.0 \Rightarrow t = \frac{1}{10}\tan^{-1}\sqrt{2.0} = 0.096\ \text{s}$$

14.38. Model: The spring undergoes simple harmonic motion.

Solve: **(a)** Total energy is $E = \frac{1}{2}kA^2$. When the displacement is $x = \frac{1}{2}A$, the potential energy is

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}k(\tfrac{1}{2}A)^2 = \tfrac{1}{4}\left(\tfrac{1}{2}kA^2\right) = \tfrac{1}{4}E \Rightarrow K = E - U = \tfrac{3}{4}E$$

One quarter of the energy is potential and three-quarters is kinetic.

(b) To have $U = \frac{1}{2}E$ requires

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}E = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \Rightarrow x = \frac{A}{\sqrt{2}}$$

14.39. Solve: Average speed is $v_{\text{avg}} = \Delta x/\Delta t$. During half a period $(\Delta t = \frac{1}{2}T)$, the particle moves from $x = -A$ to $x = +A(\Delta x = 2A)$. Thus

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{2A}{T/2} = \frac{4A}{T} = \frac{4A}{2\pi/\omega} = \frac{2}{\pi}(\omega A) = \frac{2}{\pi}v_{\max} \Rightarrow v_{\max} = \frac{\pi}{2}v_{\text{avg}}$$

14.40. Model: The ball attached to a spring is in simple harmonic motion.

Solve: **(a)** Let $t = 0$ s be the instant when $x_0 = -5.0$ cm and $v_0 = 20$ cm/s. The oscillation frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5 \text{ N/m}}{0.100 \text{ kg}}} = 5.0 \text{ rad/s}$$

Using Equation 14.27, the amplitude of the oscillation is

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-5.0 \text{ cm})^2 + \left(\frac{20 \text{ cm/s}}{5.0 \text{ rad/s}}\right)^2} = 6.4 \text{ cm}$$

(b) The maximum acceleration is $a_{\text{max}} = \omega^2 A = 160 \text{ cm/s}^2$.

(c) For an oscillator, the acceleration is most positive $(a = a_{\text{max}})$ when the displacement is most negative $(x = -x_{\text{max}} = -A)$. So the acceleration is maximum when $x = -6.4$ cm.

(d) We can use the conservation of energy between $x_0 = -5.0$ cm and $x_1 = 3.0$ cm:

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 \Rightarrow v_1 = \sqrt{v_0^2 + \frac{k}{m}\left(x_0^2 - x_1^2\right)} = 0.283 \text{ m/s}$$

The speed is 28 cm/s. Because k is known in SI units of N/m, the energy calculation *must* be done using SI units of m, m/s, and kg.

14.41. **Model:** The block on a spring is in simple harmonic motion.

Solve: **(a)** The position of the block is given by $x(t) = A\cos(\omega t + \phi_0)$. Because $x(t) = A$ at $t = 0$ s, we have $\phi_0 = 0$ rad, and the position equation becomes $x(t) = A\cos\omega t$. At $t = 0.685$ s, $3.00\text{ cm} = A\cos(0.685\omega)$ and at $t = 0.886$ s, $-3.00\text{ cm} = A\cos(0.886\omega)$. These two equations give

$$\cos(0.685\omega) = -\cos(0.886\omega) = \cos(\pi - 0.886\omega)$$
$$\Rightarrow 0.685\omega = \pi - 0.886\omega \Rightarrow \omega = 2.00\text{ rad/s}$$

(b) Substituting into the position equation,

$$3.00\text{ cm} = A\cos((2.00\text{ rad/s})(0.685\text{ s})) = A\cos(1.370) = 0.20A \Rightarrow A = \frac{3.00\text{ cm}}{0.20} = 15.0\text{ cm}$$

14.42. Model: The oscillator is in simple harmonic motion. Energy is conserved.

Solve: The energy conservation equation $E_1 = E_2$ is

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2$$

$$\frac{1}{2}(0.30\text{ kg})(0.954\text{ m/s})^2 + \frac{1}{2}k(0.030\text{ m})^2 = \frac{1}{2}(0.30\text{ kg})(0.714\text{ m/s})^2 + \frac{1}{2}k(0.060\text{ m})^2$$

$$\Rightarrow k = 44.48\text{ N/m}$$

The total energy of the oscillator is

$$E_{\text{total}} = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}(0.30\text{ kg})(0.954\text{ m/s})^2 + \frac{1}{2}(44.48\text{ N/m})(0.030\text{ m})^2 = 0.1565\text{ J}$$

Because $E_{\text{total}} = \tfrac{1}{2}mv_{\max}^2$,

$$0.1565\text{ J} = \frac{1}{2}(0.300\text{ kg})v_{\max}^2 \Rightarrow v_{\max} = 1.02\text{ m/s}$$

Assess: A maximum speed of 1.02 m/s is reasonable.

14.43. Model: The transducer undergoes simple harmonic motion.
Solve: Newton's second law for the transducer is

$$F_{\text{restoring}} = ma_{\text{max}} \Rightarrow 40{,}000\ \text{N} = \left(0.10\times10^{-3}\ \text{kg}\right)a_{\text{max}} \Rightarrow a_{\text{max}} = 4.0\times10^{8}\ \text{m/s}^2$$

Because $a_{\text{max}} = \omega^2 A$,

$$A = \frac{a_{\text{max}}}{\omega^2} = \frac{4.0\times10^{8}\ \text{m/s}^2}{\left[2\pi\left(1.0\times10^{6}\ \text{Hz}\right)\right]^2} = 1.01\times10^{-5}\ \text{m} = 10.1\ \mu\text{m}$$

(b) The maximum velocity is

$$v_{\text{max}} = \omega A = 2\pi\left(1.0\times10^{6}\ \text{Hz}\right)\left(1.01\times10^{-5}\ \text{m}\right) = 64\ \text{m/s}$$

14.44. Model: The block attached to the spring is in simple harmonic motion.
Solve: **(a)** The frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2000 \text{ N/m}}{5.0 \text{ kg}}} = 3.183 \text{ Hz}$$

The frequency is 3.2 Hz.
(b) From energy conservation,

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(0.050 \text{ m})^2 + \left(\frac{1.0 \text{ m/s}}{2\pi \cdot 3.183 \text{ Hz}}\right)^2} = 0.0707 \text{ m}$$

The amplitude is 7.1 cm.
(c) The total mechanical energy is

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(2000 \text{ N/m})(0.0707 \text{ m})^2 = 5.0 \text{ J}$$

14.45. Model: The tips of the tuning fork are in simple harmonic oscillation.

Solve: **(a)** The maximum speed is related to the amplitude.

$$v_{\max} = \omega A = 2\pi fA = 2\pi(440\ \text{Hz})(5.0\times10^{-4}\ \text{m}) = 1.38\ \text{m/s}$$

(b) The acceleration of the flea can not be greater than that allowed by the maximum force with which it can hold on. From Newton's second law, the maximum acceleration that the flea can withstand is

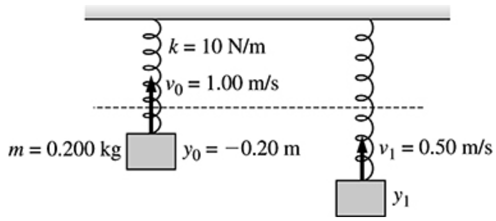
$$a_{\text{flea}} = \frac{F}{m} = \frac{1.0\times10^{-3}\ \text{N}}{10\times10^{-6}\ \text{kg}} = 100\ \text{m/s}^2$$

The maximum acceleration at the tip of the prong is

$$a_{\max} = \omega^2 A = (2\pi f)^2 A = (2\pi(440\ \text{Hz}))^2(5.0\times10^{-4}\ \text{m}) = 382\ \text{m/s}^2$$

The flea will not be able to hold on to the tuning fork.

14.46. Model: The block undergoes simple harmonic motion.
Visualize:



Solve: (a) The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10\text{ N/m}}{0.20\text{ kg}}} = 1.125\text{ Hz}$$

The frequency is 1.13 Hz.
(b) Using conservation of energy, $\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$, we find

$$x_1 = \sqrt{x_0^2 + \frac{m}{k}(v_0^2 - v_1^2)} = \sqrt{(-0.20\text{ m})^2 + \frac{0.20\text{ kg}}{10\text{ N/m}}((1.00\text{ m/s})^2 - (0.50\text{ m/s})^2)}$$
$$= 0.2345\text{ m or } 23\text{ cm}$$

(c) At time t, the displacement is $x = A\cos(\omega t + \phi_0)$. The angular frequency is $\omega = 2\pi f = 7.071\text{ rad/s}$. The amplitude is

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-0.20\text{ m})^2 + \left(\frac{1.00\text{ m/s}}{7.071\text{ rad/s}}\right)^2} = 0.245\text{ m}$$

The phase constant is

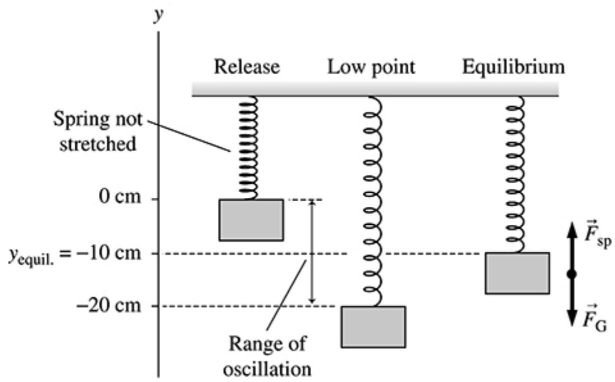
$$\phi_0 = \cos^{-1}\left(\frac{x_0}{A}\right) = \cos^{-1}\left(\frac{-0.200\text{ m}}{0.245\text{ m}}\right) = \pm 2.526\text{ rad or } \pm 145°$$

A negative displacement (below the equilibrium point) and positive velocity (upward motion) indicate that the corresponding circular motion is in the third quadrant, so $\phi_0 = -2.526$ rad. Thus at $t = 1.0$ s,

$$x = (0.245\text{ m})\cos\big((7.071\text{ rad/s})(1.0\text{ s}) - 2.526\text{ rad}\big) = -0.0409\text{ m} = -4.09\text{ cm}$$

The block is 4.1 cm below the equilibrium point.

14.47. Model: The mass is in simple harmonic motion.
Visualize:



The high point of the oscillation is *at* the point of release. This conclusion is based on energy conservation. Gravitational potential energy is converted to the spring's elastic potential energy as the mass falls and stretches the spring, then the elastic potential energy is converted 100% back into gravitational potential energy as the mass rises, bringing the mass back to *exactly* its starting height. The total displacement of the oscillation—high point to low point—is 20 cm. Because the oscillations are symmetrical about the equilibrium point, we can deduce that the *equilibrium* point of the spring is 10 cm below the point where the mass is released. The mass oscillates about this equilibrium point with an amplitude of 10 cm, that is, the mass oscillates between 10 cm above and 10 cm below the equilibrium point.

Solve: The equilibrium point is the point where the mass would hang *at rest*, with $F_{\text{sp}} = F_{\text{G}} = mg$. At the equilibrium point, the spring is stretched by $\Delta y = 10\text{ cm} = 0.10\text{ m}$. Hooke's law is $F_{\text{sp}} = k\Delta y$, so the equilibrium condition is

$$\left[F_{\text{sp}} = k\Delta y\right] = \left[F_{\text{G}} = mg\right] \Rightarrow \frac{k}{m} = \frac{g}{\Delta y} = \frac{9.8\text{ m/s}^2}{0.10\text{ m}} = 98\text{ s}^{-2}$$

The *ratio* k/m is all we need to find the oscillation frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{98\text{ s}^{-2}} = 1.58\text{ Hz}$$

14.48. Model: The spring is ideal, so the apples undergo SHM.
Solve: The spring constant of the scale can be found by considering how far the pan goes down when the apples are added.

$$\Delta L = \frac{mg}{k} \Rightarrow k = \frac{mg}{\Delta L} = \frac{20 \text{ N}}{0.090 \text{ m}} = 222 \text{ N/m}$$

The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{222 \text{ N/m}}{(20 \text{ N}/9.8 \text{ m/s}^2)}} = 1.66 \text{ Hz}$$

Assess: An oscillation of fewer than twice per second is reasonable.

14.49. Model: The compact car is in simple harmonic motion.

Solve: **(a)** The mass on each spring is $(1200 \text{ kg})/4 = 300 \text{ kg}.$ The spring constant can be calculated as follows:

$$\omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2 = m(2\pi f)^2 = (300 \text{ kg})\left[2\pi(2.0 \text{ Hz})\right]^2 = 4.74\times 10^4 \text{ N/m}$$

The spring constant is 4.7×10^4 N/m.

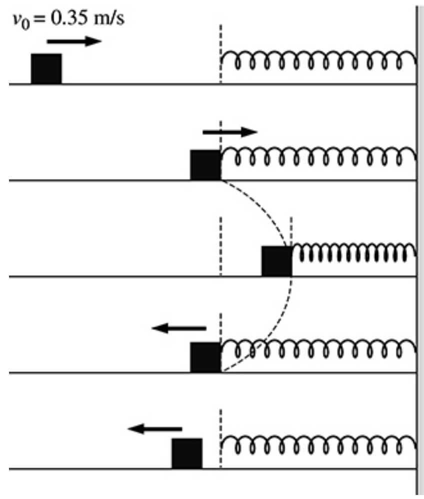
(b) The car carrying four persons means that each spring has, on the average, an additional mass of 70 kg. That is, $m = 300 \text{ kg} + 70 \text{ kg} = 370 \text{ kg}.$ Thus,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.74\times 10^4 \text{ N/m}}{370 \text{ kg}}} = 1.80 \text{ Hz}$$

Assess: A small frequency change from the additional mass is reasonable because frequency is inversely proportional to the square root of the mass.

14.50. Model: Hooke's law for the spring. The spring's compression and decompression constitutes simple harmonic motion.

Visualize:



Solve: **(a)** The spring's compression or decompression is one-half of the oscillation cycle. This means the contact time is $\Delta t = \frac{1}{2}T,$ where T is the period. The period is calculated as follows:

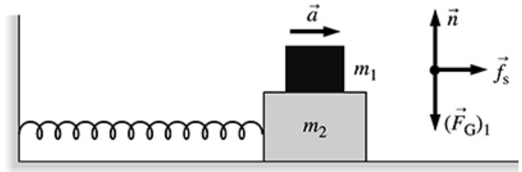
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\ \text{N/m}}{0.500\ \text{kg}}} = 10\ \text{rad/s} \Rightarrow T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{10\ \text{rad/s}} = 0.628\ \text{s}$$

$$\Rightarrow \Delta t = \frac{T}{2} = 0.31\ \text{s}$$

(b) There is no change in contact time, because period of oscillation is independent of the amplitude or the maximum speed.

14.51. Model: The two blocks are in simple harmonic motion, without the upper block slipping. We will also apply the model of static friction between the two blocks.

Visualize:



Solve: The net force acting on the upper block m_1 is the force of friction due to the lower block m_2. The model of static friction gives the maximum force of static friction as

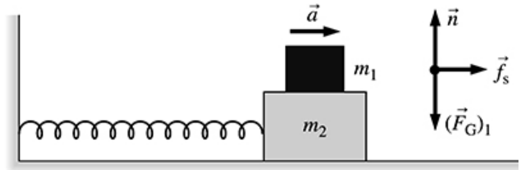
$$(f_s)_{max} = \mu_s n = \mu_s(m_1 g) = m_1 a_{max} \Rightarrow a_{max} = \mu_s g$$

Using $\mu_s = 0.5$, $a_{max} = \mu_S g = (0.5)(9.8\ \text{m/s}^2) = 4.9\ \text{m/s}^2$. That is, the two blocks will ride together if the maximum acceleration of the system is equal to or less than a_{max}. We can calculate the maximum value of A as follows:

$$a_{max} = \omega^2 A_{max} = \frac{k}{m_1 + m_2} A_{max} \Rightarrow A_{max} = \frac{a_{max}(m_1 + m_2)}{k} = \frac{(4.9\ \text{m/s}^2)(1.0\ \text{kg} + 5.0\ \text{kg})}{50\ \text{N/m}} = 0.59\ \text{m}$$

14.52. Model: Assume simple harmonic motion for the two-block system without the upper block slipping. We will also use the model of static friction between the two blocks.

Visualize:



Solve: The net force on the upper block m_1 is the force of static friction due to the lower block m_2. The two blocks ride together as long as the static friction doesn't exceed its maximum possible value. The model of static friction gives the maximum force of static friction as

$$(f_s)_{max} = \mu_s n = \mu_s(m_1 g) = m_1 a_{max} \Rightarrow a_{max} = \mu_s g$$

$$\Rightarrow \mu_s = \frac{a_{max}}{g} = \frac{\omega^2 A_{max}}{g} = \left(\frac{2\pi}{T}\right)^2\left(\frac{A_{max}}{g}\right) = \left(\frac{2\pi}{1.5 \text{ s}}\right)^2\left(\frac{0.40 \text{ m}}{9.8 \text{ m/s}^2}\right) = 0.72$$

Assess: Because the period is given, we did not need to use the block masses or the spring constant in our calculation.

14.53. Model: The DNA and cantilever undergo SHM.
Visualize: Please refer to figure P14.53.
Solve: The cantilever has the same spring constant with and without the DNA molecule. The frequency of oscillation without the DNA is

$$\omega_1 = \sqrt{\frac{k}{\frac{1}{3}M}}$$

With the DNA, the frequency of oscillation is

$$\omega_2 = \sqrt{\frac{k}{\frac{1}{3}M + m}}$$

where m is the mass of the DNA.
Divide the two equations, and express $\omega_2 = \omega_1 - \Delta\omega$, where $\Delta\omega = 2\pi\Delta f = 2\pi(50 \text{ Hz})$.

$$\frac{\omega_1}{\omega_2} = \frac{f_1}{f_2} = \frac{f_1}{f_1 - \Delta f} = \sqrt{\frac{\frac{k}{(\frac{1}{3}M)}}{\frac{k}{(\frac{1}{3}M + m)}}} = \sqrt{\frac{\frac{1}{3}M + m}{\frac{1}{3}M}}$$

Thus

$$f_1^2\left(\tfrac{1}{3}M\right) = \left(f_1 - \Delta f\right)^2\left(\tfrac{1}{3}M + m\right)$$

$$m = \tfrac{1}{3}M\frac{\left(f_1^2 - \left(f_1 - \Delta f\right)^2\right)}{\left(f_1 - \Delta f\right)^2} = \tfrac{1}{3}M\left(\frac{f_1^2}{\left(f_1 - \Delta f\right)^2} - 1\right)$$

Since $\Delta f \ll f_1$, $(50 \text{ Hz} \ll 12\text{MHz})$, $\left(f_1 - \Delta f\right)^{-2} = f_1^{-2}\left(1 - \frac{\Delta f}{f_1}\right)^{-2} \approx f_1^{-2}\left(1 + 2\frac{\Delta f}{f_1}\right)$. Thus

$$m = \tfrac{1}{3}M\left(1 + 2\frac{\Delta f}{f_1} - 1\right) = \tfrac{2}{3}M\frac{\Delta f}{f_1}$$

The mass of the cantilever

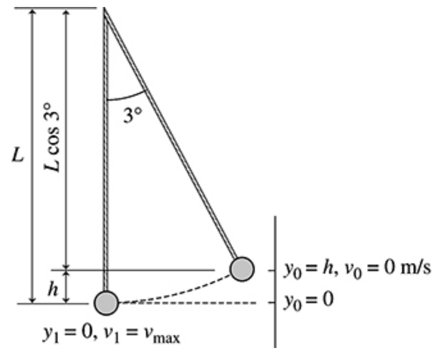
$$M = (2300 \text{ kg/m}^3)(4000\times10^{-9} \text{ m})(100\times10^{-9} \text{ m}) = 3.68\times10^{-16} \text{ kg}$$

Thus the mass of the DNA molecule is

$$m = \frac{2}{3}\left(3.68\times10^{-16} \text{ kg}\right)\left(\frac{50 \text{ Hz}}{12\times10^6 \text{ Hz}}\right) = 1.02\times10^{-21} \text{ kg}$$

Assess: The mass of the DNA molecule is about 6.2×10^5 atomic mass units, which is reasonable for such a large molecule.

14.54. Model: Assume that the swinging lamp makes a small angle with the vertical so that there is simple harmonic motion.
Visualize:



Solve: **(a)** Using the formula for the period of a pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = g\left(\frac{T}{2\pi}\right)^2 = \left(9.8\ \text{m/s}^2\right)\left(\frac{5.5\ \text{s}}{2\pi}\right)^2 = 7.5\ \text{m}$$

(b) The conservation of mechanical energy equation $K_0 + U_{g0} = K_1 + U_{g1}$ for the swinging lamp is

$$\tfrac{1}{2}mv_0^2 + mgy_0 = \tfrac{1}{2}mv_1^2 + mgy_1 \Rightarrow 0\ \text{J} + mgh = \tfrac{1}{2}mv_{max}^2 + 0\ \text{J}$$

$$\Rightarrow v_{max} = \sqrt{2gh} = \sqrt{2g\left(L - L\cos 3°\right)}$$

$$= \sqrt{2\left(9.8\ \text{m/s}^2\right)\left(7.5\ \text{m}\right)\left(1 - \cos 3°\right)} = 0.45\ \text{m/s}$$

14.55. Model: Assume that the angle with the vertical that the pendulum makes is small enough so that there is simple harmonic motion.

Solve: The angle θ made by the string with the vertical as a function of time is

$$\theta(t)=\theta_{\max}\cos(\omega t+\phi_0)$$

The pendulum starts from maximum displacement, thus $\phi_0=0$. Thus, $\theta(t)=\theta_{\max}\cos\omega t$. To find the time t when the pendulum reaches 4.0° on the opposite side:

$$(-4.0°)=(8.0°)\cos\omega t\Rightarrow\omega t=\cos^{-1}(-0.5)=2.094\text{ rad}$$

Using the formula for the angular frequency,

$$\omega=\sqrt{\frac{g}{L}}=\sqrt{\frac{9.8\text{ m/s}^2}{1.0\text{ m}}}=3.130\text{ rad/s}\Rightarrow t=\frac{2.0944\text{ rad}}{\omega}=\frac{2.094\text{ rad}}{3.130\text{ rad/s}}=0.669\text{ s}$$

The time $t=0.67$ s.

Assess: Because $T=2\pi/\omega=2.0$ s, a value of 0.67 s for the pendulum to cover a little less than half the oscillation is reasonable.

14.56. Model: Assume a small angle oscillation of the pendulum so that it has simple harmonic motion.
Solve: **(a)** At the equator, the period of the pendulum is

$$T_{\text{equator}} = 2\pi\sqrt{\frac{1.000\ \text{m}}{9.78\ \text{m/s}^2}} = 2.009\ \text{s}$$

The time for 100 oscillations is 200.9 s.
(b) At the north pole, the period is

$$T_{\text{pole}} = 2\pi\sqrt{\frac{1.000\ \text{m}}{9.83\ \text{m/s}^2}} = 2.004\ \text{s}$$

The time for 100 oscillations is 200.4 s.
(c) The difference between the two answers is 0.5 s, and this difference is quite measurable with a hand-operated stopwatch.
(d) The period on the top of the mountain is 2.010 s. The acceleration due to gravity can be calculated by rearranging the formula for the period:

$$g_{\text{mountain}} = L\left(\frac{2\pi}{T_{\text{mountain}}}\right)^2 = (1.000\ \text{m})\left(\frac{2\pi}{2.010\ \text{s}}\right)^2 = 9.772\ \text{m/s}^2$$

Assess: This last result is reasonable because g decreases with altitude.

14.57. Model: The mass is a particle and the string is massless.

Solve: Equation 14.52 is

$$\omega = \sqrt{\frac{Mgl}{I}}$$

The moment of inertia of the mass on a string is $I = Ml^2$, where l is the length of the string. Thus

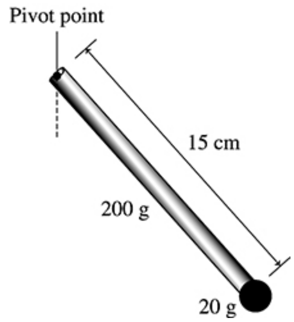
$$\omega = \sqrt{\frac{Mgl}{Ml^2}} = \sqrt{\frac{g}{l}}$$

This is Equation 14.49 with $L = l$.

Assess: Equation 14.49 is really a specific case of the more general physical pendulum described by Equation 14.52.

14.58. Model: The rod is thin and uniform with moment of inertia described in Table 12.2. The clay ball is a particle located at the end of the rod. The ball and rod together form a physical pendulum. The oscillations are small.

Visualize:



Solve: The moment of inertia of the composite pendulum formed by the rod and clay ball is

$$I_{\text{rod + ball}} = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}m_{\text{rod}}L^2 + m_{\text{ball}}L^2$$

$$= \frac{1}{3}(0.200 \text{ kg})(0.15 \text{ m})^2 + (0.020 \text{ kg})(0.15 \text{ m})^2 = 1.95\times10^{-3} \text{ kg m}^2$$

The center of mass of the rod and ball is located at a distance from the pivot point of

$$y_{\text{cm}} = \frac{(0.200 \text{ kg})\left(\frac{0.15}{2} \text{ m}\right) + (0.020 \text{ kg})(0.15 \text{ m})}{(0.200 \text{ kg} + 0.020 \text{ kg})} = 8.18\times10^{-2} \text{ m}$$

The frequency of oscillation of a physical pendulum is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgl}{I}} = \frac{1}{2\pi}\sqrt{\frac{(0.220 \text{ kg})(9.8 \text{ m/s}^2)(8.18\times10^{-2} \text{ m})}{1.95\times10^{-3} \text{ kg m}^2}} = 1.51 \text{ Hz}$$

The period of oscillation $T = \dfrac{1}{f} = 0.66$ s.

14.59. Model: The circular hoop can be modeled as a cylindrical hoop and its moment of inertia about the point of rotation found with the parallel-axis theorem.
Visualize: Please refer to Figure P14.59.
Solve: Using the parallel-axis theorem, the moment of inertia of the cylindrical hoop about the rotation point is

$$I = MR^2 + MR^2 = 2MR^2$$

The frequency of small oscillations is given by Equation 14.52.

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgl}{I}}$$

The center of mass of the hoop is its center, so $l = R$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{MgR}{2MR^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$$

14.60. Model: The motion is a damped oscillation.

Solve: The position of the air-track glider is $x(t) = Ae^{-(t/2\tau)}\cos(\omega t + \phi_0)$, where $\tau = m/b$ and

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Using $A = 0.20$ m, $\phi_0 = 0$ rad, and $b = 0.015$ kg/s,

$$\omega = \sqrt{\frac{4.0\ \text{N/m}}{0.250\ \text{kg}} - \frac{(0.015\ \text{kg/s})^2}{4(0.250\ \text{kg})^2}} = \sqrt{16 - 9\times10^{-4}}\ \text{rad/s} = 4.0\ \text{rad/s}$$

Thus the period is

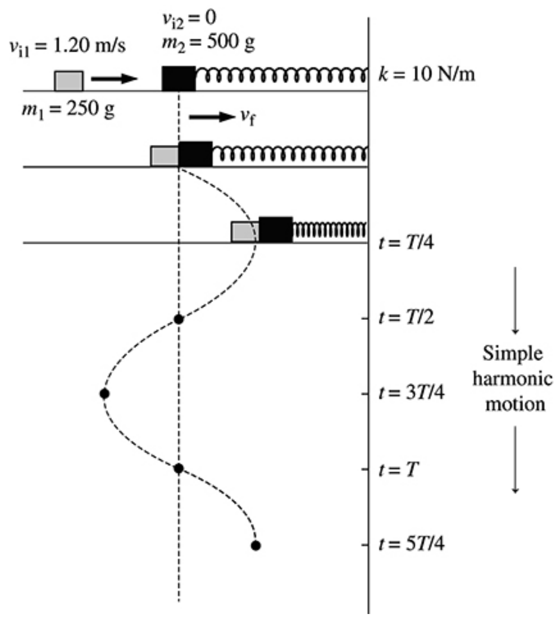
$$T = \frac{2\pi}{\omega} = \frac{2\pi\ \text{rad}}{4.0\ \text{rad/s}} = 1.57\ \text{s}$$

The amplitude at $t = 0$ s is $x_0 = A$ and the amplitude will be equal to $e^{-1}A$ at a time given by

$$\frac{1}{e}A = Ae^{-(t/2\tau)} \Rightarrow t = 2\tau = 2\frac{m}{b} = 33.3\ \text{s}$$

The number of oscillations in a time of 33.3 s is $(33.3\ \text{s})/(1.57\ \text{s}) = 21$.

14.61. Model: A completely inelastic collision between the two gliders resulting in simple harmonic motion.
Visualize:



Let us denote the 250 g and 500 g masses as m_1 and m_2, which have initial velocities v_{i1} and v_{i2}. After m_1 collides with and sticks to m_2, the two masses move together with velocity v_f.

Solve: The momentum conservation equation $p_f = p_i$ for the completely inelastic collision is $(m_1 + m_2)v_f = m_1 v_{i1} + m_2 v_{i2}$. Substituting the given values,

$$(0.750 \text{ kg})v_f = (0.250 \text{ kg})(1.20 \text{ m/s}) + (0.500 \text{ kg})(0 \text{ m/s}) \Rightarrow v_f = 0.400 \text{ m/s}$$

We now use the conservation of mechanical energy equation:

$$(K + U_s)_{\text{compressed}} = (K + U_s)_{\text{equilibrium}} \Rightarrow 0 \text{ J} + \tfrac{1}{2}kA^2 = \tfrac{1}{2}(m_1 + m_2)v_f^2 + 0 \text{ J}$$

$$\Rightarrow A = \sqrt{\frac{m_1 + m_2}{k}}v_f = \sqrt{\frac{0.750 \text{ kg}}{10 \text{ N/m}}}(0.400 \text{ m/s}) = 0.110 \text{ m}$$

The period is

$$T = 2\pi\sqrt{\frac{m_1 + m_2}{k}} = 2\pi\sqrt{\frac{0.750 \text{ kg}}{10 \text{ N/m}}} = 1.72 \text{ s}$$

14.62. Model: The block attached to the spring is oscillating in simple harmonic motion.

Solve: **(a)** Because the frequency of an object in simple harmonic motion is independent of the amplitude and/or the maximum velocity, the new frequency is equal to the old frequency of 2.0 Hz.

(b) The speed v_0 of the block just before it is given a blow can be obtained by using the conservation of mechanical energy equation as follows:

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}mv_0^2$$

$$\Rightarrow v_0 = \sqrt{\frac{k}{m}}A = \omega A = (2\pi f)A = (2\pi)(2.0\text{ Hz})(0.02\text{ m}) = 0.25\text{ m/s}$$

The blow to the block provides an impulse that changes the velocity of the block:

$$J_x = F_x\Delta t = \Delta p = mv_\text{f} - mv_0$$

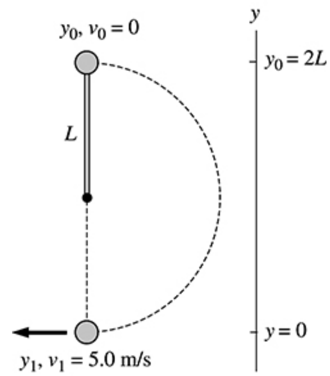
$$(-20\text{ N})(1.0\times10^{-3}\text{ s}) = (0.200\text{ kg})v_\text{f} - (0.200\text{ kg})(0.25\text{ m/s}) \Rightarrow v_\text{f} = 0.150\text{ m/s}$$

Since v_f is the new maximum velocity of the block at the equilibrium position, it is equal to $A\omega$. Thus,

$$A = \frac{0.150\text{ m/s}}{\omega} = \frac{0.150\text{ m/s}}{2\pi(2.0\text{ Hz})} = 0.012\text{ m} = 1.19\text{ cm}$$

Assess: Because v_f is positive, the block continues to move to the right even after the blow.

14.63. Model: The pendulum falls, then undergoes small-amplitude oscillations in simple harmonic motion.
Visualize:



We placed the origin of the coordinate system at the bottom of the arc.
Solve: We need to find the length of the pendulum. The conservation of mechanical energy equation for the pendulum's fall is $(K+U_g)_{top} = (K+U_g)_{bottom}$:

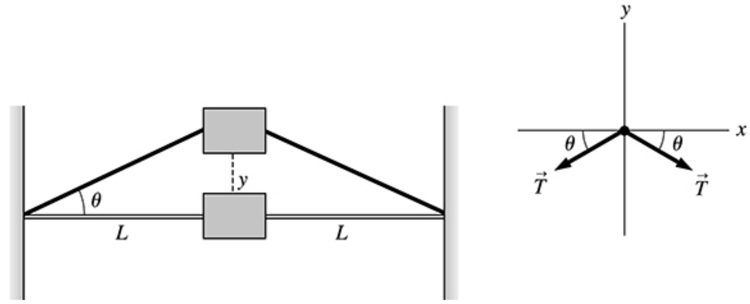
$$\tfrac{1}{2}mv_0^2 + mgy_0 = \tfrac{1}{2}mv_1^2 + mgy_1 \Rightarrow 0\text{ J} + mg(2L) = \tfrac{1}{2}m(5.0\text{ m/s})^2 + 0\text{ J}$$

$$\Rightarrow L = \frac{1}{4}\frac{(5.0\text{ m/s})^2}{g} = 0.6377\text{ m}$$

Using $L = 0.6377$ m, we can find the frequency f as

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\text{ m/s}^2}{0.6377\text{ m}}} = 0.62\text{ Hz}$$

14.64. Model: Assume the small-angle approximation.
Visualize:



Solve: The tension in the two strings pulls downward at angle θ. Thus Newton's second law is

$$\sum F_y = -2T\sin\theta = ma_y$$

From the geometry of the figure we can see that

$$\sin\theta = \frac{y}{\sqrt{L^2 + y^2}}$$

If the oscillation is small, then $y \ll L$ and we can approximate $\sin\theta \approx y/L$. Since y/L is $\tan\theta$, this approximation is equivalent to the small-angle approximation $\sin\theta \approx \tan\theta$ if $\theta \ll 1$ rad. With this approximation, Newton's second law becomes

$$-2T\sin\theta \approx -\frac{2T}{L}y = ma_y = m\frac{d^2y}{dt^2} \quad \Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{2T}{mL}y$$

This is the equation of motion for simple harmonic motion (see Equations 14.33 and 14.47). The constants $2T/mL$ are equivalent to k/m in the spring equation or g/L in the pendulum equation. Thus the oscillation frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{2T}{mL}}$$

14.65. Visualize: Please refer to Figure P14.65.

Solve: The potential energy curve of a simple harmonic oscillator is described by $U = \frac{1}{2}k(\Delta x)^2$, where $\Delta x = x - x_0$ is the displacement from equilibrium. From the graph, we see that the equilibrium bond length is $x_0 =$ 0.13 nm. We can find the bond's spring constant by reading the value of the potential energy U at a displacement Δx and using the potential energy formula to calculate *k*.

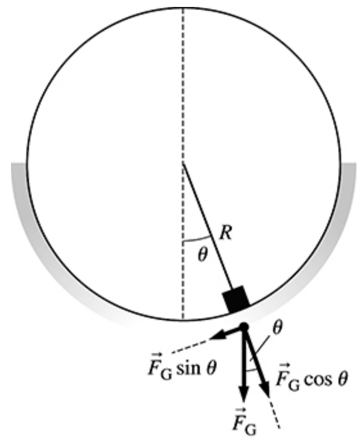
x (nm)	Δx (nm)	U (J)	k (N/m)
0.11	0.02	0.8×10^{-19} J	400
0.10	0.03	1.9×10^{-19} J	422
0.09	0.04	3.4×10^{-19} J	425

The three values of *k* are all very similar, as they should be, with an average value of 416 N/m. Knowing the spring constant, we can now calculate the oscillation frequency of a hydrogen atom on this "spring" to be

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{416 \text{ N/m}}{1.67 \times 10^{-27} \text{ kg}}} = 7.9 \times 10^{13} \text{ Hz}$$

14.66. Model: Assume that the size of the ice cube is much less than R and that θ is a small angle.

Visualize:



Solve: The ice cube is like an object on an inclined plane. The net force on the ice cube in the tangential direction is

$$-(F_G)\sin\theta = ma = mR\alpha = mR\frac{d^2\theta}{dt^2} \Rightarrow -mg\sin\theta = mR\frac{d^2\theta}{dt^2}$$

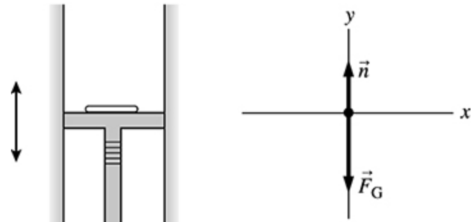
where α is the angular acceleration. With the small-angle approximation $\sin\theta \approx \theta,$ this becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{R}\theta = -\omega^2\theta$$

This is the equation of motion of an object in simple harmonic motion with a period of

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

14.67. Visualize:



Free-body diagram for the penny

Solve: **(a)** Newton's second law applied to the penny along the *y*-axis is

$$F_{net} = n - mg = ma_y$$

$\vec{F}_{net}$ is upward at the bottom of the cycle (positive a_y), so $n > mg$. The speed is maximum when passing through equilibrium, but $a_y = 0$ so $n = mg$. The critical point is the *highest* point. $\vec{F}_{net}$ points down and a_y is negative. If a_y becomes sufficiently negative, n drops to zero and the penny is no longer in contact with the surface.

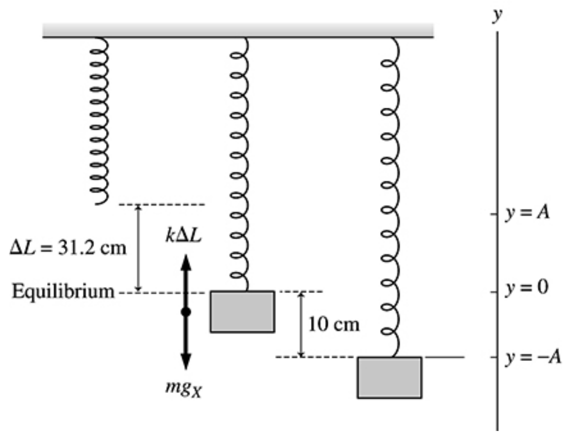
(b) When the penny loses contact ($n = 0$), the equation for Newton's law becomes $a_{max} = g$. For simple harmonic motion,

$$a_{max} = A\omega^2 \Rightarrow \omega = \sqrt{\frac{g}{A}} = \sqrt{\frac{9.8\ \text{m/s}^2}{0.040\ \text{m}}} = 15.65\ \text{rad}$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{15.65\ \text{rad/s}}{2\pi} = 2.5\ \text{Hz}$$

14.68. Model: The vertical oscillations constitute simple harmonic motion.

Visualize:



Solve: At the equilibrium position, the net force on mass m on Planet X is:

$$F_{\text{net}} = k\Delta L - mg_{\text{X}} = 0\ \text{N} \Rightarrow \frac{k}{m} = \frac{g_{\text{X}}}{\Delta L}$$

For simple harmonic motion $k/m = \omega^2$, thus

$$\omega^2 = \frac{g_{\text{X}}}{\Delta L} \Rightarrow \omega = \sqrt{\frac{g_{\text{X}}}{\Delta L}} = \frac{2\pi}{T} \Rightarrow g_{\text{X}} = \left(\frac{2\pi}{T}\right)^2 \Delta L = \left(\frac{2\pi}{14.5\ \text{s}/10}\right)^2 (0.312\ \text{m}) = 5.86\ \text{m/s}^2$$

14.69. Model: The doll's head is in simple harmonic motion and is damped.

Solve: (a) The oscillation frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = m(2\pi f)^2 = (0.015\ \text{kg})(2\pi)^2(4.0\ \text{Hz})^2 = 9.475\ \text{N/m}$$

The spring constant is 9.5 N/m.

(b) The maximum speed is

$$v_{\text{max}} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{9.475\ \text{N/m}}{0.015\ \text{kg}}}(0.020\ \text{m}) = 0.50\ \text{m/s}$$

(c) Using $A(t) = A_0 e^{-bt/2m}$, we get

$$(0.5\ \text{cm}) = (2.0\ \text{cm})e^{-b(4.0\ \text{s})/(2\times 0.015\ \text{kg})} \Rightarrow 0.25 = e^{-(133.3\ \text{s/kg})b}$$

$$\Rightarrow -(133.33\ \text{s/kg})b = \ln 0.25 \Rightarrow b = 0.0104\ \text{kg/s}$$

14.70. Model: The oscillator is in simple harmonic motion.

Solve: **(a)** The maximum displacement at time *t* of a damped oscillator is

$$x_{\max}(t) = Ae^{-t/2\tau} \Rightarrow -\frac{t}{2\tau} = \ln\left(\frac{x_{\max}(t)}{A}\right)$$

Using $x_{\max} = 0.98A$ at $t = 0.50$ s, we can find the time constant τ to be

$$\tau = -\frac{0.50 \text{ s}}{2\ln(0.98)} = 12.375 \text{ s}$$

25 oscillations will be completed at $t = 25T = 12.5$ s. At that time, the amplitude will be

$$x_{\max,\ 12.5 \text{ s}} = (10 \text{ cm})e^{-12.5 \text{ s}/(2)(12.375 \text{ s})} = 6.0 \text{ cm}$$

(b) The energy of a damped oscillator decays more rapidly than the amplitude: $E(t) = E_0 e^{-1/\tau}$. When the energy is 60% of its initial value, $E(t)/E_0 = 0.60$. We can find the time this occurs as follows:

$$-\frac{t}{\tau} = \ln\left(\frac{E(t)}{E_0}\right) \Rightarrow t = -\tau\ln\left(\frac{E(t)}{E_0}\right) = -(12.375 \text{ s})\ln(0.60) = 6.3 \text{ s}$$

14.71. Model: The oscillator is in simple harmonic motion.

Solve: The maximum displacement, or amplitude, of a damped oscillator decreases as $x_{max}(t) = Ae^{-t/2\tau}$, where τ is the time constant. We know $x_{max}/A = 0.60$ at $t = 50$ s, so we can find τ as follows:

$$-\frac{t}{2\tau} = \ln\left(\frac{x_{max}(t)}{A}\right) \Rightarrow \tau = -\frac{50 \text{ s}}{2\ln(0.60)} = 48.9 \text{ s}$$

Now we can find the time t_{30} at which $x_{max}/A = 0.30$:

$$t_{30} = -2\tau\ln\left(\frac{x_{max}(t)}{A}\right) = -2(48.9 \text{ s})\ln(0.30) = 118 \text{ s}$$

The undamped oscillator has a frequency $f = 2 \text{ Hz} = 2$ oscillations per second. Damping changes the oscillation frequency slightly, but the text notes that the change is negligible for “light damping.” Damping by air, which allows the oscillations to continue for well over 100 s, is certainly light damping, so we will use $f = 2.0$ Hz. Then the number of oscillations before the spring decays to 30% of its initial amplitude is

$$N = f \cdot t_{30} = (2 \text{ oscillations/s}) \cdot (118 \text{ s}) = 236 \text{ oscillations}$$

14.72. Solve: The solution of the equation

$$\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=0$$

is $x(t)=Ae^{-bt/2m}\cos(\omega t+\phi_0)$. The first and second derivatives of $x(t)$ are

$$\frac{dx}{dt}=-\frac{Ab}{2m}e^{-bt/2m}\cos(\omega t+\phi_0)-A\omega e^{-bt/2m}\sin(\omega t+\phi_0)$$

$$\frac{d^2x}{dt^2}=\left[\left(\frac{Ab^2}{4m^2}-A\omega^2\right)\cos(\omega t+\phi_0)+\frac{\omega Ab}{m}\sin(\omega t+\phi_0)\right]e^{-bt/2m}$$

Substituting these expressions into the differential equation, the terms involving $\sin(\omega t+\phi_0)$ cancel and we obtain the simplified result

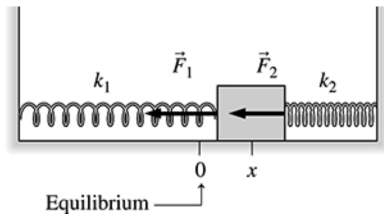
$$\left(\frac{-b^2}{4m^2}-\omega^2+\frac{k}{m}\right)\cos(\omega t+\phi_0)=0$$

Because $\cos(\omega t+\phi_0)$ is not equal to zero in general,

$$\frac{-b^2}{4m^2}-\omega^2+\frac{k}{m}=0\Rightarrow\omega=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$$

14.73. Model: The two springs obey Hooke's law.

Visualize:



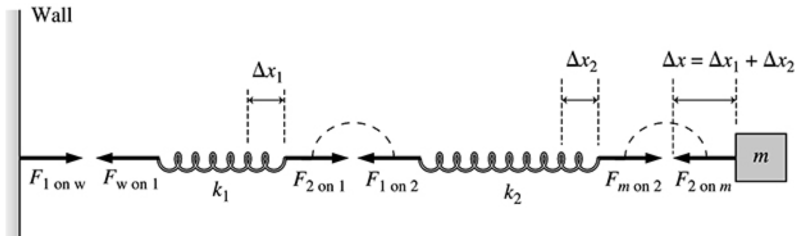
Solve: There are *two* restoring forces on the block. If the block's displacement x is positive, *both* restoring forces—one pushing, the other pulling—are directed to the left and have negative values:

$$(F_{\text{net}})_x = (F_{\text{sp 1}})_x + (F_{\text{sp 2}})_x = -k_1x - k_2x = -(k_1 + k_2)x = -k_{\text{eff}}x$$

where $k_{\text{eff}} = k_1 + k_2$ is the effective spring constant. This means the oscillatory motion of the block under the influence of the two springs will be the *same* as if the block were attached to a single spring with spring constant k_{eff}. The frequency of the blocks, therefore, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{k_1}{4\pi^2 m} + \frac{k_2}{4\pi^2 m}} = \sqrt{f_1^2 + f_2^2}$$

14.74. Model: The two springs obey Hooke's law. Assume massless springs.
Visualize: Each spring is shown separately. Note that $\Delta x = \Delta x_1 + \Delta x_2$.



Solve: Only spring 2 touches the mass, so the net force on the mass is $F_m = F_{2 \text{ on } m}$. Newton's third law tells us that $F_{2 \text{ on } m} = F_{m \text{ on } 2}$ and that $F_{2 \text{ on } 1} = F_{1 \text{ on } 2}$. From $F_{\text{net}} = ma$, the net force on a massless spring is zero. Thus $F_{\text{w on } 1} = F_{2 \text{ on } 1} = k_1 \Delta x_1$ and $F_{m \text{ on } 2} = F_{1 \text{ on } 2} = k_2 \Delta x_2$. Combining these pieces of information,

$$F_m = k_1 \Delta x_1 = k_2 \Delta x_2$$

The net displacement of the mass is $\Delta x = \Delta x_1 + \Delta x_2$, so

$$\Delta x = \Delta x_1 + \Delta x_2 = \frac{F_m}{k_1} + \frac{F_m}{k_2} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right) F_m = \frac{k_1 + k_2}{k_1 k_2} F_m$$

Turning this around, the net force on the mass is

$$F_m = \frac{k_1 k_2}{k_1 + k_2} \Delta x = k_{\text{eff}} \Delta x \quad \text{where } k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}$$

k_{eff}, the proportionality constant between the force on the mass and the mass's displacement, is the effective spring constant. Thus the mass's angular frequency of oscillation is

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{1}{m} \frac{k_1 k_2}{k_1 + k_2}}$$

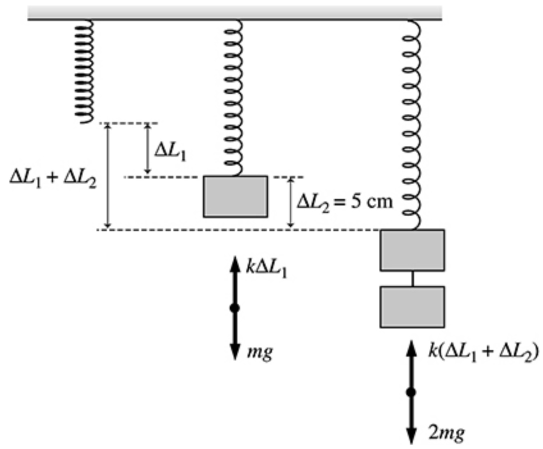
Using $\omega_1^2 = k_1/m$ and $\omega_2^2 = k_2/m$ for the angular frequencies of either spring acting alone on m, we have

$$\omega = \sqrt{\frac{(k_1/m)(k_2/m)}{(k_1/m) + (k_2/m)}} = \sqrt{\frac{\omega_1^2 \omega_2^2}{\omega_1^2 + \omega_2^2}}$$

Since the actual frequency f is simply a multiple of ω, this same relationship holds for f:

$$f = \sqrt{\frac{f_1^2 f_2^2}{f_1^2 + f_2^2}}$$

14.75. Model: The blocks undergo simple harmonic motion.
Visualize:



The length of the stretched spring due to a block of mass m is ΔL_1. In the case of the two block system, the spring is further stretched by an amount ΔL_2.

Solve: The equilibrium equations from Newton's second law for the single block and double block systems are

$$(\Delta L_1)k = mg \text{ and } (\Delta L_1 + \Delta L_2)k = (2m)g$$

Using $\Delta L_2 = 5.0$ cm, and subtracting these two equations, gives us

$$(\Delta L_1 + \Delta L_2)k - \Delta L_1 k = (2m)g - mg \Rightarrow (0.05 \text{ m})k = mg$$

$$\Rightarrow \quad \frac{k}{m} = \frac{9.8 \text{ m/s}^2}{0.05 \text{ m}} = 196 \text{ s}^{-2}$$

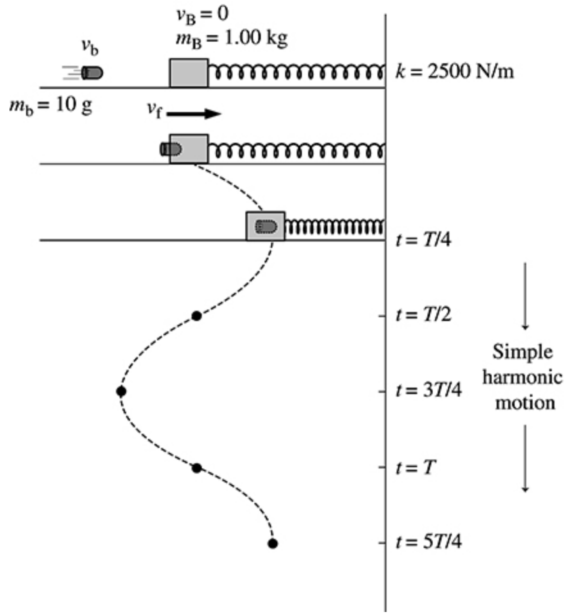
With both blocks attached, giving total mass $2m$, the angular frequency of oscillation is

$$\omega = \sqrt{\frac{k}{2m}} = \sqrt{\frac{1}{2}\frac{k}{m}} = \sqrt{\frac{1}{2}196 \text{ s}^{-2}} = 9.90 \text{ rad/s}$$

Thus the oscillation frequency is $f = \omega/2\pi = 1.58$ Hz.

14.76. Model: A completely inelastic collision between the bullet and the block resulting in simple harmonic motion.

Visualize:



Solve: **(a)** The equation for conservation of energy after the collision is

$$\frac{1}{2}kA^2 = \frac{1}{2}(m_b + m_B)v_f^2 \Rightarrow v_f = \sqrt{\frac{k}{m_b + m_B}}A = \sqrt{\frac{2500\ \text{N/m}}{1.010\ \text{kg}}}(0.10\ \text{m}) = 5.0\ \text{m/s}$$

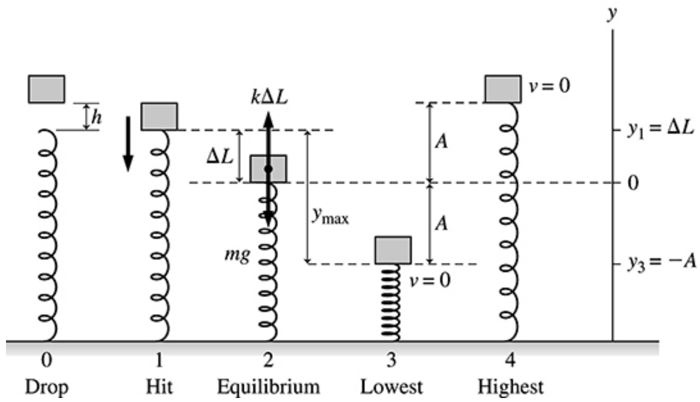
The momentum conservation equation for the perfectly inelastic collision $p_{after} = p_{before}$ is

$$(m_b + m_B)v_f = m_b v_b + m_B v_B$$
$$(1.010\ \text{kg})(5.0\ \text{m/s}) = (0.010\ \text{kg})v_b + (1.00\ \text{kg})(0\ \text{m/s}) \Rightarrow v_b = 5.0\times10^2\ \text{m/s}$$

(b) No. The oscillation frequency $\sqrt{k/(m_b + m_B)}$ depends on the masses but not on the speeds.

14.77. Model: The block undergoes SHM after sticking to the spring. Energy is conserved throughout the motion.

Visualize:



It's essential to carefully visualize the motion. At the highest point of the oscillation the spring is *stretched* upward.

Solve: We've placed the origin of the coordinate system at the equilibrium position, where the block would sit on the spring at rest. The spring is compressed by ΔL at this point. Balancing the forces requires $k\Delta L = mg$. The angular frequency is $w^2 = k/m = g/\Delta L$, so we can find the oscillation frequency by finding ΔL. The block hits the spring (1) with kinetic energy. At the lowest point (3), kinetic energy and gravitational potential energy have been transformed into the spring's elastic energy. Equate the energies at these points:

$$K_1 + U_{1g} = U_{3s} + U_{3g} \Rightarrow \tfrac{1}{2}mv_1^2 + mg\Delta L = \tfrac{1}{2}k(\Delta L + A)^2 + mg(-A)$$

We've used $y_1 = \Delta L$ as the block hits and $y_3 = -A$ at the bottom. The spring has been compressed by $\Delta y = \Delta L + A$. Speed v_1 is the speed after falling distance h, which from free-fall kinematics is $v_1^2 = 2gh$. Substitute this expression for v_1^2 and $mg/\Delta L$ for k, giving

$$mgh + mg\Delta L = \frac{mg}{2(\Delta L)}(\Delta L + A)^2 + mg(-A)$$

The mg term cancels, and the equation can be rearranged into the quadratic equation

$$(\Delta L)^2 + 2h(\Delta L) - A^2 = 0$$

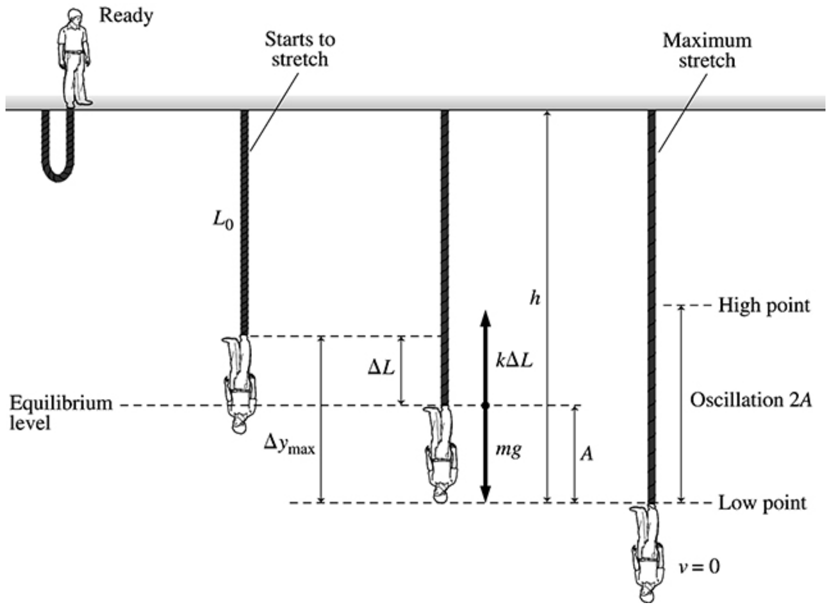
The positive solution is

$$\Delta L = \sqrt{h^2 + A^2} - h = \sqrt{(0.030\text{ m})^2 + (0.100\text{ m})^2} - 0.030\text{ m} = 0.0744\text{ m}$$

Now that ΔL is known, we can find

$$\omega = \sqrt{\frac{g}{\Delta L}} = \sqrt{\frac{9.80\text{ m/s}^2}{0.0744\text{ m}}} = 11.48\text{ rad/s} \Rightarrow f = \frac{\omega}{2\pi} = 1.83\text{ Hz}$$

14.78. Model: Model the bungee cord as a spring. The motion is damped SHM.
Visualize:



Solve: **(a)** For light damping, the oscillation period is

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = m\left(\frac{2\pi}{T}\right)^2 = (75\text{ kg})\left(\frac{2\pi}{4.0\text{ s}}\right)^2 = 185\text{ N/m}$$

(b) The maximum speed is $v_{max} = \omega A = (2\pi/4.0\text{ s})(11.0\text{ m}) = 17.3\text{ m/s}$.

(c) Jose oscillates about the equilibrium position at which he would hang at rest. Balancing the forces, $\Delta L = mg/k =$ (75 kg)(9.80 m/s^2)(185 N/m) = 3.97 m. Jose's lowest point is 11.0 m below this point, so the bungee cord is stretched by $\Delta y_{max} = \Delta L + A = 14.97$ m. Choose this lowest point as $y = 0$. Because Jose is instantaneously at rest at this point, his energy is entirely the elastic potential energy of the stretched bungee cord. Initially, his energy was entirely gravitational potential energy. Equating his initial energy to his energy at the lowest point,

$$U_{\text{lowest point}} = U_{\text{highest point}} \Rightarrow \tfrac{1}{2}k(\Delta y_{max})^2 = mgh$$

$$h = \frac{k(\Delta y_{max})^2}{2mg} = \frac{(185\text{ N/m})(14.97\text{ m})^2}{2(75\text{ kg})(9.80\text{ m/s}^2)} = 28.2\text{ m}$$

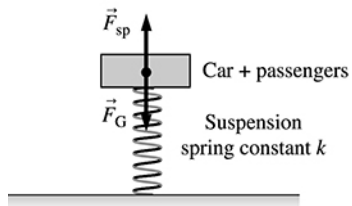
Jose jumped 28 m above the lowest point.

(d) The amplitude decreases due to damping as $A(t) = Ae^{-bt/2\text{m}}$. At the time when the amplitude has decreased from 11.0 m to 2.0 m,

$$\frac{2.0\text{ m}}{11.0\text{ m}} = e^{-bt/2m} \Rightarrow t = -\frac{2m}{b}\ln\left(\frac{2}{11}\right) = -\frac{2(75\text{ kg})}{6.0\text{ kg/s}}(-1.705) = 42.6\text{ s}$$

With a period of 4.0 s, the number of oscillations is $N_{osc} = (42.6\text{ s})/(4.0\text{ s}) = 10.7$ oscillations.

14.79. Model: The vertical movement of the car is simple harmonic motion.
Visualize:



The fact that the car has a *maximum* oscillation amplitude at 5 m/s implies a *resonance.* The bumps in the road provide a periodic external force to the car's suspension system, and a resonance will occur when the "bump frequency" f_{ext} matches the car's natural oscillation frequency f_0.

Solve: Now the 5.0 m/s is not a frequency, but we can convert it to a frequency because we know the bumps are spaced every 3.0 meters. The time to drive 3.0 m at 5.0 m/s is the period:

$$T = \frac{\Delta x}{v} = \frac{3.0 \text{ m}}{5.0 \text{ m/s}} = 0.60 \text{ s}$$

The external frequency due to the bumps is thus $f_{ext} = 1/T = 1.667$ Hz. This matches the car's natural frequency f_0, which is the frequency the car oscillates up and down if you push the car down and release it. This is enough information to deduce the spring constant of the car's suspension:

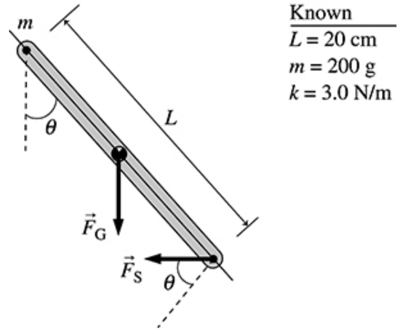
$$1.667 \text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = m\left(2\pi f_{ext}\right)^2 = 131{,}600 \ \frac{\text{N}}{\text{m}}$$

where we used $m = m_{total} = m_{car} + 2m_{passenger} = 1200$ kg. When at rest, the car is in static equilibrium with $F_{net} = 0$ N. The downward weight $m_{total}g$ of the car and passengers is balanced by the upward spring force $k\Delta y$ of the suspension. Thus the compression Δy of the suspension is

$$\Delta y = \frac{m_{total}g}{k}$$

Initially $m_{total} = m_{car} + 2m_{passenger} = 1200$ kg, causing an initial compression $\Delta y_i = 0.0894 \text{ m} = 8.94 \text{ cm}$. When three additional passengers get in, the mass increases to $m_{total} = m_{car} + 5m_{passenger} = 1500$ kg. The final compression is $\Delta y_f = 0.1117 \text{ m} = 11.17 \text{ cm}$. Thus the three new passengers cause the suspension to "sag" by $11.17 \text{ cm} - 8.94 \text{ cm} = 2.23 \text{ cm}$.

14.80. Model: The rod is thin and uniform.
Visualize: Please refer to Figure CP14.80.



Solve: We must derive our own equation for this combination of a pendulum and spring. For small oscillations, $\vec{F}_s$ remains horizontal. The net torque around the pivot point is

$$\tau_{net} = I\alpha = -F_s L\cos\theta - F_G\left(\frac{L}{2}\right)\sin\theta$$

With $\alpha = \dfrac{d^2\theta}{dt^2}$, $F_G = mg$, $F_s = k\Delta x = kL\sin\theta$, and $I = \dfrac{1}{3}mL^2$,

$$\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\sin\theta\cos\theta - \frac{3g}{2L}\sin\theta$$

We can use $\sin\theta\cos\theta = \dfrac{1}{2}\sin 2\theta$. For small angles, $\sin\theta \approx \theta$ and $\sin 2\theta \approx 2\theta$. So

$$\frac{d^2\theta}{dt^2} = -\left(\frac{3k}{m} + \frac{3g}{2L}\right)\theta$$

This is the same as Equations 14.33 and 14.47 with

$$\omega = \sqrt{\frac{3k}{m} + \frac{3g}{2L}}$$

The frequency of oscillation is thus

$$f = \frac{1}{2\pi}\sqrt{\frac{3(3.0\text{ N/m})}{(0.200\text{ kg})} + \frac{3(9.8\text{ m/s}^2)}{2(0.20\text{ m})}} = 1.73\text{ Hz}$$

The period $T = \dfrac{1}{f} = 0.58$ s.

Assess: Fewer than two oscillations per second is reasonable. The rod's angle from the vertical must be small enough that $\sin 2\theta \approx 2\theta$. This is more restrictive than other examples, which only require that $\sin\theta \approx \theta$.